Attribute Sampling Plans

August 12, 2020

Outline

Single Sampling Plan

A single sampling plan for inspecting a lot of N items consists of

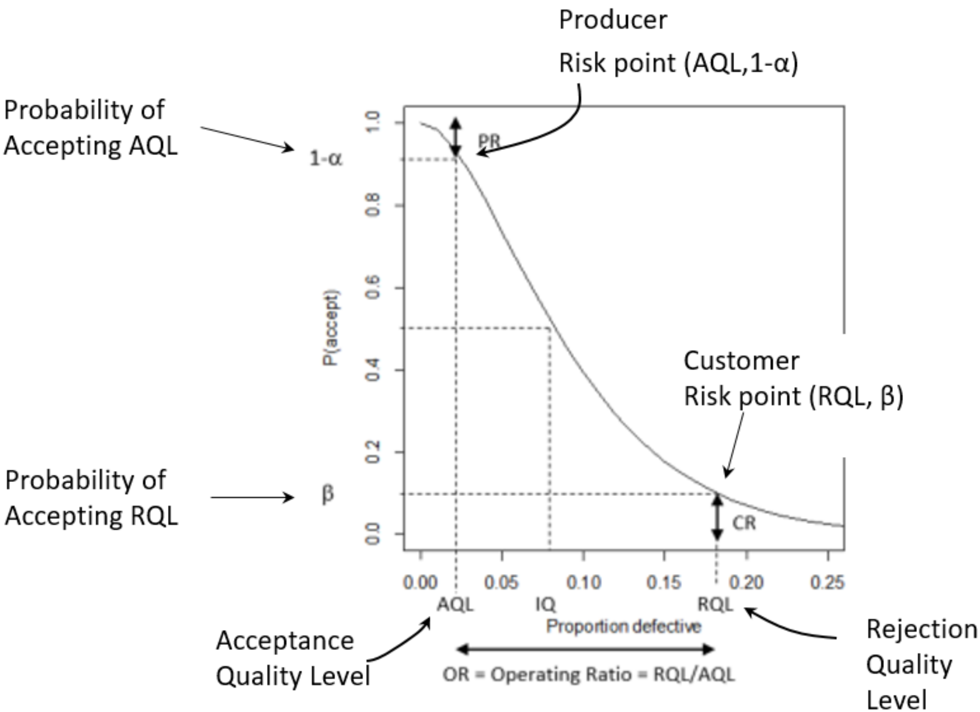
- the number of items to sample n from the lot of N items.
- the maximum number of nonconforming items in the lot, c, that will allow the lot to be accepted.

The operating Characteristic Curve for a single sampling plan

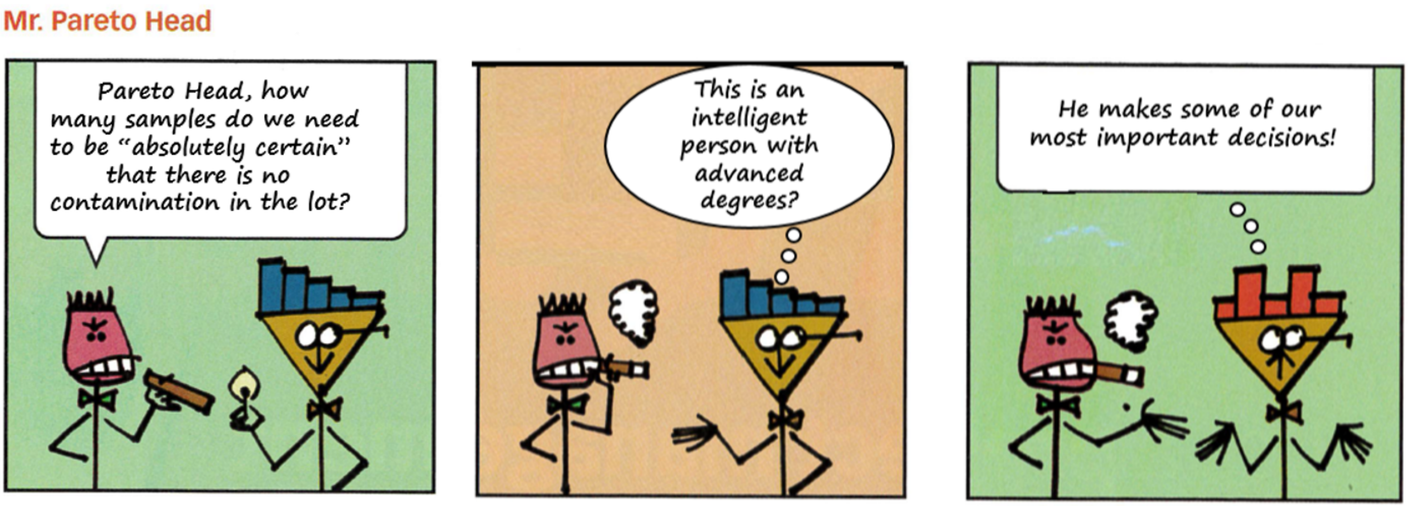
The operating characteristic OC is the probability of accepting a lot with proportion nonconforming $p = D/N$. It is calculated using the Hypergeometric distribution as shown below.

$$OC = Pr(accept|p = D/N) = \sum_{i=0}^{c} \frac{\binom{D}{i}\binom{N-D}{n-i}}{\binom{N}{n}} \tag{1}$$

The OC or probability of acceptance is a function of p the proportion non-conforming in the lot or D the number nonconforming in the lot. The OC curve shown on the next slide graphs the probability of acceptance on the y-axis versus the proportion non-conforming on the x-axis.

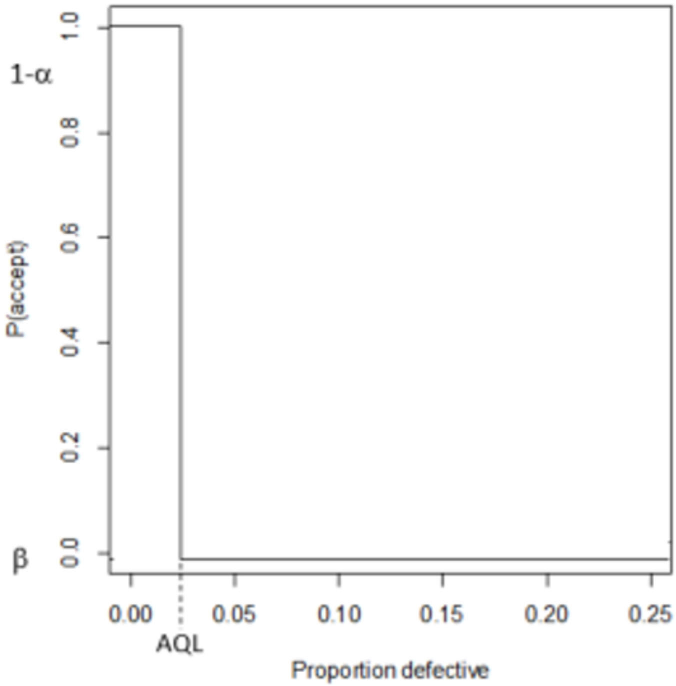


The Acceptance Quality Level (AQL) represents a proportion non-conforming that is acceptable to the customer. The Rejection Quality Level (RQL) represents a proportion non-conforming that is undesirable to the customer. 1-α should be a high probability like .95 or greater, and β should be a low probability of .10 or less.



The only way to guarantee RQL=$1/N$ has zero probability of being accepted would be to do 100% inspection of the lot!

Ideal OC Curve with α=0, and β=0



This is impossible unless

$n = N$ 100% inspection,

and $c = N \times \text{AQL}$

$$P(\text{Accept} = 1.0 \mid p \leq AQL)$$

$$P(\text{Accept} = 0.0 \mid p \geq AQL)$$

Acceptance Sampling Package in R

$$Pr(accept) = \sum_{i=0}^{c} \frac{\binom{D}{i}\binom{N-D}{n-i}}{\binom{N}{n}}$$

The **find.plan** function attempts to find n and c so that the probability of accepting when $D = 0.05 \times N$ is as close to 1-0.05=.95 as possible, and the probability of accepting when $D = 0.15 \times N$ is as close to 0.20 as possible.

```
library(AcceptanceSampling)
find.plan(PRP=c(0.05,0.95),CRP=c(0.15,0.20),type="hypergeom",N=500)
```

Producer risk point | AQL | 1-α | Customer risk point | RQL | β

```
## $n
## [1] 51
##
## $c
## [1] 5
##
## $r
## [1] 6
```

Type A and B OC Curves

For a large lot size relative to the sample size Probability of accepting a Lot of N items with D non-conforming

$$Pr(accept) = \sum_{i=0}^{c} \frac{\binom{D}{i}\binom{N-D}{n-i}}{\binom{N}{n}} \sim \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$$

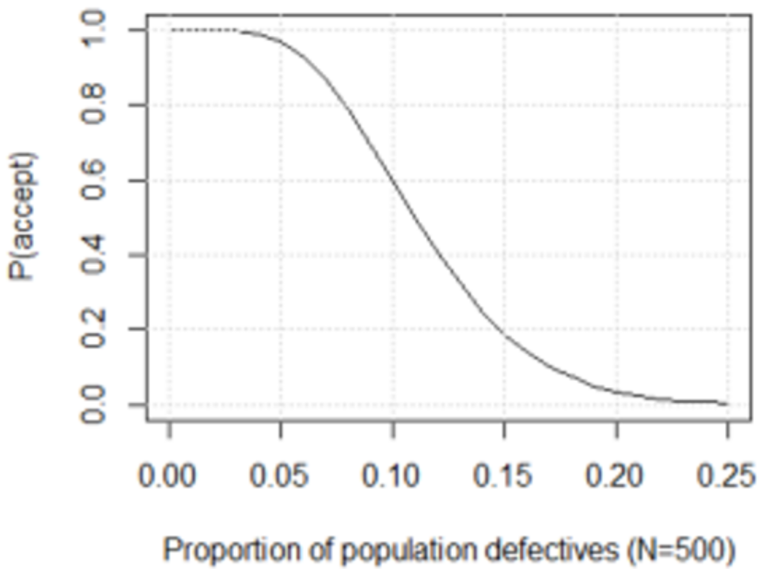
Type A OC curve   Type B OC curve

```
library(AcceptanceSampling)
find.plan(PRP=c(0.05,0.95),CRP=c(0.15,0.20),type="hypergeom",N=500)
```

Can be "binomial" for large N

Plotting the OC Curve

```
library(AcceptanceSampling)
plan<-OC2c(51,5,type="hypergeom", N=500, pd=seq(0,.25,.01))
plot(plan, type='l')
grid()
```



Assessing how well a plan meets a specified PRP and CRP

```
assess(OC2c(51,5), PRP=c(0.05, 0.95), CRP=c(0.15,0.20))


Acceptance Sampling Plan (binomial)

               Sample 1
Sample size(s)       51
Acc. Number(s)        5
Rej. Number(s)        6

Plan CANNOT meet desired risk point(s):

           Quality   RP P(accept) Plan P(accept)
PRP           0.05           0.95      0.9589318
CRP           0.15           0.20      0.2032661
```

Double and Multiple Sampling Plans

A Double Sampling Plan consists of sample sizes n_1 and n_2, acceptance numbers c_1 and c_2, and rejection numbers r_1 and r_2, where $r_1 > c_1 + 2$. To execute the plan take a sample of n_1 items, if there are c_1 or fewer items nonconforming then accept the lot. If there are r_1 or more items nonconforming reject the lot. If the number of nonconforming items $c_1 < nc < r_1$ then take a second sample of n_2 items. If after the second sample the combined number of nonconforming items is less than or equal to c_2 accept the lot. If the combined number of nonconforming items is greater or equal to r_2 then reject the lot. For most all double sampling plans $r_2 = c_2 + 1$ so that a decision must be made by the second sample.

Multiple Sampling Plan

A Multiple Sampling Plan can be represented as shown in the table below

Table 2.1: A Multiple Sampling Plan

Sample	Sample Size	Cum. Samp. Size	Acc. Number	Rej. Number
1	n_1	n_1	c_1	r_1
2	n_2	$n_1 + n_2$	c_2	r_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	n_k	$n_1 + n_2 + \ldots + n_k$	c_k	$r_k = c_k + 1$

Accept after the ith sample if the combined number of nonconforming items in the samples is less than or equal to c_i. Reject if the combined number of nonforororming items is greater or equal to r_i otherwise take another Sample.

Comparing Single and Double Sampling Plans

Double Sampling plans can be found that will have essentially the same OC Curve as a particular single sampling plan. Consider the single sampling plan used for lots of $N = 1000$ with $n = 134$, $c = 3$, $r = 4$.

```
plns<-OC2c(n=134,c=3,type="hypergeom", N=1000,pd=seq(0,.20,.01))
assess(plns,PRP=c(.01,.95),CRP=c(.05,.10))
Acceptance Sampling Plan (hypergeom)


               Sample 1
Sample size(s)      134
Acc. Number(s)        3
Rej. Number(s)        4


Plan CAN meet desired risk point(s):


           Quality   RP P(accept) Plan P(accept)
PRP           0.01           0.95     0.96615674
CRP           0.05           0.10     0.07785287
```

Comparing Single and Double Sampling Plans

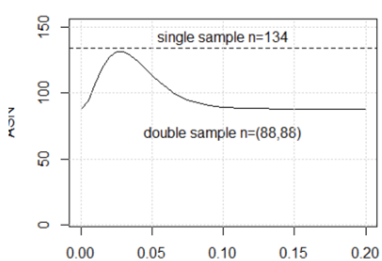
A Double sampling Plan with essentially the same OC Curve has $n_1 = 88$, $n_2 = 88$, $c_1 = 1$, $c_2 = 4$, $r_1 = 4$, $r_2 = 5$

```
pln3<-OC2c(n=c(88,88),c=c(1,4),r=c(4,5),type="hypergeom",N=1000,pd=seq(0,.20,.01))
assess(pln3,PRP=c(.01,.95),CRP=c(.05,.10))
Acceptance Sampling Plan (hypergeom)


               Sample 1 Sample 2
Sample size(s)       88       88
Acc. Number(s)        1        4
Rej. Number(s)        4        5


Plan CAN meet desired risk point(s):


           Quality   RP P(accept) Plan P(accept)
PRP           0.01           0.95      0.9805612
CRP           0.05           0.10      0.0776524
```

Comparing Single and Double Sampling Plans

The average sample number for the double sampling plan is given by $ASN = n_1 + n_2 \times P(c_1 < x_1 < r_1)$ and it is uniformly below the sample size for the OC equivalent single sampling plan

```
D=seq(0,200,5) #Number nonconforming in the lot of 1000
pd<-D/1000     #Proportion nonconforming in the lot of 1000
OC1<-phyper(c(1), m=D, n=1000-D, k=88, lower.tail=TRUE)
#Probability of accepting after the first sample
R1<-phyper(c(3), m=D, n=1000-D, k=88, lower.tail=FALSE)
#Probability of rejecting after the first sample
P<-OC1+R1
ASN=88+88*(1-P)
plot(pd,ASN,type='l',ylim=c(5,150),xlab="Proportion nonconforming in the lot")
abline(134,0,lty=2)
text(.10,142,'single sample n=134')
text(.10,70,'double sample n=(88,88)')
grid()
```



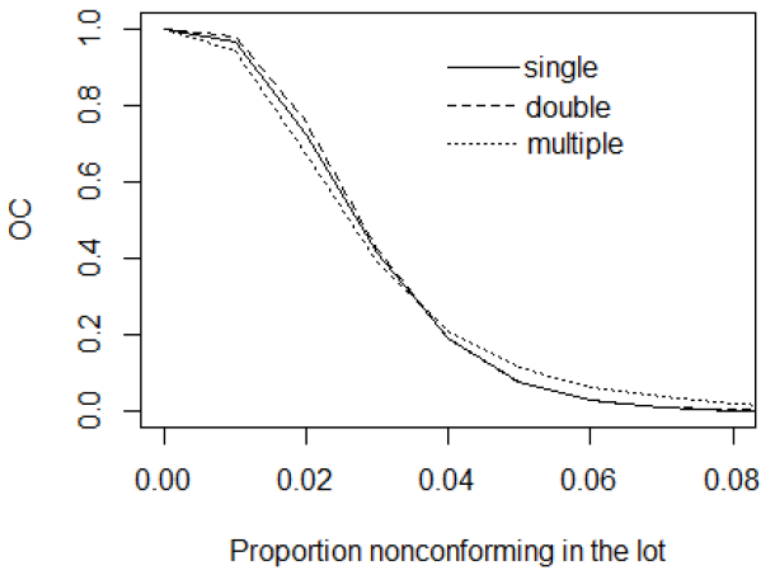
Comparing Single Double and Multiple Plans

A multiple sampling plan can also be found with an equivalent OC curve it is shown in the table below

Sample	Sample Size	Cum. Samp. Size	Acc. Number	Rej. Number
1	46	46	0	3
2	46	92	1	3
3	46	138	2	4
4	46	184	3	5
5	46	230	4	6
6	46	276	6	7

Comparing Single Double and Multiple Plans

The OC curves for the single, double and multiple plans are nearly the same



However, the average sample number for the multiple plan will be uniformly below that for the single and double plan.

Rectification Inspection

When rectification sampling is used, every nonconforming item found in the sample is replaced with a conforming item. Further, if the lot is rejected, the remaining items in the lot are also inspected, and any additional nonconforming items found are replaced by conforming items. In that way every rejected lot will be 100% inspected and all nonconforming items replaced with conforming items.

Rectification Inspection

Assuming that an ongoing stream of lots is being inspected where the the probability of a nonconforming item in the suppliers process is p, then the probability of accepting a lot by the single sampling plan(n, c) is given by the Binomial distribution

$$OC = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{2}$$

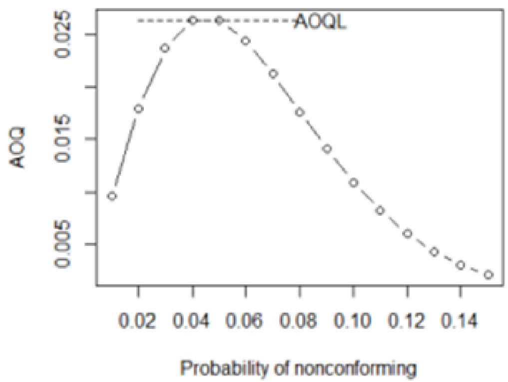
and the average outgoing quality level is

$$AOQ = \frac{OC \times p(N-n)}{N} \tag{3}$$

AOQ Curve and AOQL

The AOQ is a function of the p the proportion nonconforming in the suppliers process, and maximum AOQ as a function of p is called the $AOQL$ or average outgoing quality limit. The R code below calculates and graphs these quantities.

```
library(AcceptanceSampling)
p<-seq(.01,.15,.01)
N<-2000
n<-50
c<-2
Pa<-pbinom(c,n,p,lower.tail=TRUE)
AOQ<-(Pa*p*(N-n))/N
plot(p,AOQ,type='b',xlab='Probability of nonconforming')
lines(c(.02,.08),c(.02639,.02639),lty=2)
text(.088,.02639,"AOQL")
```



ATI and ASN for Single or Double Sampling with Rectification

For a single sampling plan with rectification, the number of items inspected is either n or N, and and the average total inspection (ATI) required is:

$$ATI = n + (1 - P_a)(N - n).$$

If rectification is used with a double sampling plan,

$$ASN = n_1(P_{a_1} + P_{r_1}) + n_2(1 - P_{a_1} - P_{r_1})$$

$$ATI = n_1 P_{a_1} + (n_1 + n_2)P_{a_2} + N(1 - P_{a_1} - P_{a_2})$$

where n_1 is the sample size for the first sample, n_2 is the sample size for the second sample, P_{a_1} is the probability of accepting on the first sample, P_{a_2} is the probability of accepting on the second sample given by $\sum_{i=c_1+1}^{r_1-1} P(x_1 = i)P(x_2 \leq c_2 - x_1)$ where x_1 is the number nonconforming on the first sample and x_2 is the number nonconforming on the second sample.

Dodge-Romig Rectification Plans

Dodge and Romig developed tables for rectification plans in the late 1920s and early 1930s at the Bell Telephone Laboratories. Their tables were first published in the Bell System Technical Journal and later in book form (Dodge and Romig 1959). They provided both single and double sampling for attributes.

There are two sets of plans. One set minimized the ATI for various values of the LTPD. The other set of tables minimize the ATI for a specified level of AOQL protection. The LTPD based tables are useful when you want to specify an LTPD protection on each lot inspected. The AOQL based plans are useful to guarantee the outgoing quality levels regardless of the quality coming to the inspection station. The tables for each set of plans require that the supplier's process average percent nonconforming be known. If this is not known, it can be entered as the highest level shown in the table to get a conservative plan.

Dodge-Romig Rectification Plans

The LTPD based plans provide more protection on individual lots and therefore require higher sample sizes than the AOQL based plans. While the book by (Christensen, Betz, and Stein 2013) does not contain the Dodge Romig tables, they are available in the book by (Montgomery 2013) and the online calculator at https://www.sqconline.com/ will provide the single sample plans for both AOQL and LTPD protection.

When used by a supply process within the same company as the customer, a more recent and better way of insuring that the average proportion nonconforming is low is to use the quality management techniques and statistical process control techniques advocated in MIL-STD-1916. Statistical process control techniques will be discussed in Chapters 4, 5, and 6.

Dodge-Romig Rectification Plans

As an example of retrieving a Dodge-Romig AOQL plan using the online calculator at https://www.sqconline.com/, go to the website log in and click ion the Dodge-Romig Single Sampling AOQL calulator as shown below:

Calculator	What is it for?
MIL-STD-105E (ANSI/ASQ Z1.4) ISO 2859-1 (BS 6001-1)	Sampling plans for attribute (pass/fail) data
MIL-STD-414 ANSI/ASQC Z1.9, ISO 3951-1, BS 6002	Sampling plans for measurement data
MIL-STD-1235C Procedure CSP-1	Sampling plans for continuous production
Dodge-Romig Single Sampling AOQL	AOQL-based rectifying plan for attribute (pass/fail) data
Dodge-Romig Single Sampling LTPD	LTPD-based rectifying plan for attribute (pass/fail) data
MIL-STD-1916 for Attributes	Accept-on-Zero (c=0) sampling plans for attribute (pass/fail) data
MIL-STD-1916 for Variables	Accept-on-Zero (c=0) sampling plans for measurement data
MIL-STD-1916 for Continuous Sampling	Accept-on-Zero (c=0) sampling plans for continuous production
Squeglia Zero-Based Plans	AQL-based Accept-on-Zero (c=0) sampling plans for attribute (pass/fail) data

Dodge-Romig Rectification Plans

Next, enter the lot size, process percent nonconforming, and desired AOQL as shown below.

Dodge-Romig Sampling Inspection Tables (BETA)

Single Sampling for Stated Values of Average Outgoing Quality Limit (AOQL)

This application gives rectifying sampling plans for attributes.

Enter your process parameters:

Lot size	250	The number of items in the batch. Must be no larger than 100,000.
AOQL	4.00% ▾	
Process Average (%)	1.6 %	Must be between 0 and AOQL

Submit

Dodge-Romig Rectification Plans

The resulting plan is shown below.

Dodge-Romig Sampling Inspection Tables (BETA)

Single Sampling for Stated Values of Average Outgoing Quality Limit (AOQL)

This application gives rectifying sampling plans for attributes.

For a lot of **250** items, with AOQL **4.00%**, and process average **1.6%**, the inspection plan is:

Sample **20** items.

If the number of non-conforming items is:

- 0 → accept the lot
- 1 → rectify the non-conforming item and accept the lot
- 2 or more → inspect the entire lot and rectify all non-conforming items.

Sampling Schemes

When a customer company expects to receive ongoing shipments of lots from a trusted supplier, instead of one isolated lot, it is better to base the OC curve on the Binomial Distribution, and it is better to use a scheme of acceptance sampling plans (rather than one plan) to inspect the incoming stream of lots. When a sampling scheme is utilized, there is more than one sampling plan and switching rules to dictate which sampling plan should be used at a particular time. The switching rules are based on previous samples.

When basing the OC curve on the Binomial Distribution, the OC or probability of acceptance is given by:

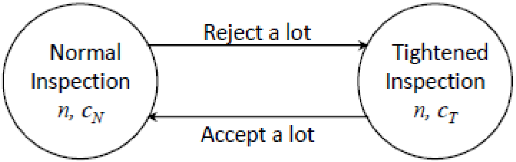
$$Pr(accept) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{4}$$

Quick Switching Scheme

Romboski(1969) proposed a straightforward sampling scheme called the quick switching scheme QSS-1. His plan proceeds as follows. There are two acceptance sampling plans. One is called the normal inspection plan consisting of sample size n and acceptance number c_N, the second is called the tightened inspection plan consisting of the same sample size n, but a reduced acceptance number c_T. The following switching rules are used:

1. Start using the normal inspection plan.
2. Switch to the tightened inspection plan immediately following a rejected lot.
3. When on tightened inspection, switch back to normal inspection immediately following an accepted lot.
4. Alternate back and forth based on these rules.

This plan can be diagrammed simply as shown below



Quick Switching Scheme

The scheme can be viewed as a two state Markov chain with the two states being normal inspection and tightened inspection. Based on this fact, Romboski(1969) determined that the OC or probability of acceptance of a lot by the scheme (or combination of the two plans) was given by:

$$Pr(accept) = \frac{P_T}{(1 - P_N) + P_T}, \tag{5}$$

where:

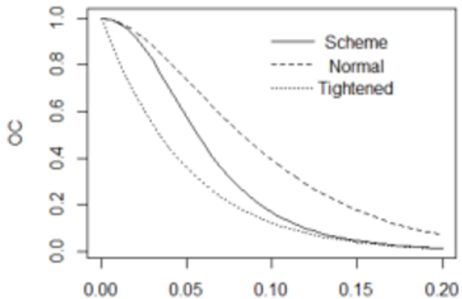
$$P_N = \sum_{i=0}^{c_N} \binom{n}{i} p^i (1-p)^{n-i}, \tag{6}$$

$$P_T = \sum_{i=0}^{c_T} \binom{n}{i} p^i (1-p)^{n-i}. \tag{7}$$

Quick Switching Scheme

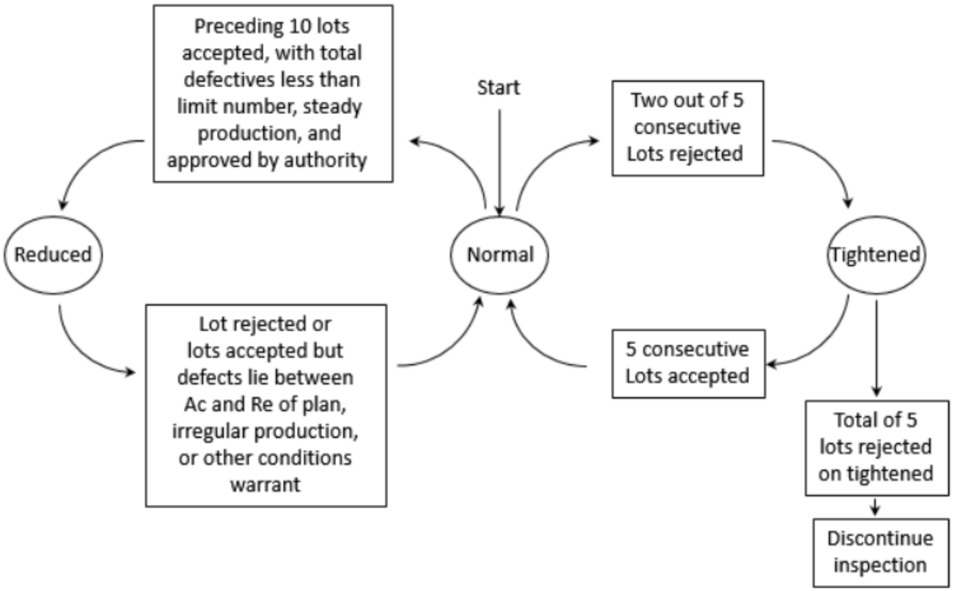
For the quick switching scheme with n=20, c_N=1, and c_T=0, the R code below uses equations on the last slide to calculate the OC curves for the scheme, normal, and tightened inspection, and then creates a graph to compare them.

```
# Comparison of Normal, Tightened, and QSS-1 OC Curves
pd<-seq(0,.20,.005)
PN=pbinom(1,20,pd)
PT=pbinom(0,20,pd)
Pa<-PT/((1-PN)+PT)
plot(pd,Pa,type='l',lty=1,xlim=c(0,.2),xlab='Probability of a nonconforming',ylab="OC")
lines(pd,PN,type='l',lty=2)
lines(pd,PT,type='l',lty=3)
lines(c(.10,.125),c(.9,.9),lty=1)
lines(c(.10,.125),c(.8,.8),lty=2)
lines(c(.10,.125),c(.7,.7),lty=3)
text(.15,.9,'Scheme')
text(.15,.8,'Normal')
text(.15,.7,'Tightened')
```



ANSI/ASQ Z1.4 Attribute Sampling Scheme

ANSI/ASQ Standard Z1.4 is the American national standard derived from MIL-STD-105E. It incorporates minor changes, none of which involved the central tables. It is recommended by the Department of Defense as a replacement and is best used for domestic contracts and in house-use. A diagram of the switching rules is shown below.



ANSI/ASQ Z1.4 Single Sampling Plan

An ANSI/ASQ Single Sampling plan for a lot size of $N = 1750$, with normal sampling, AQL=1.5%, and inspection level II, can be retrieved using the `AQLSchemes` package as shown below:

```
> library(AQLSchemes)
> planS<-AASingle('Normal')

What is the Inspection Level?

1: S-1
2: S-2
3: S-3
4: S-4
5: I
6: II
7: III

Selection: 6

What is the Lot Size?

 1: 2-8                2: 9-15               3: 16-25              4: 26-50
 5: 51-90              6: 91-150             7: 151-280            8: 281-500
 9: 501-1200          10: 1201-3200         11: 3201-10,000       12: 10,001-35,000
13: 35,001-150,000    14: 150,001-500,000   15: 500,001 and over

Selection: 10
```

ANSI/ASQ Z1.4 Single Sampling Plan

```
What is the AQL in percent nonconforming per 100 items?

 1: 0.010   2: 0.015   3: 0.025   4: 0.040   5: 0.065   6: 0.10    7: 0.15    8: 0.25
 9: 0.40   10: 0.65   11: 1.0    12: 1.5    13: 2.5    14: 4.0    15: 6.5    16: 10
17: 15     18: 25     19: 40     20: 65     21: 100    22: 150    23: 250    24: 400
25: 650    26: 1000

Selection: 12

> planS
    n c r
1 125 5 6
```

Here it can be seen that the plan calls for a sample on $n = 125$ with an acceptance number of $c = 5$ and a rejection number of $r = 6$. Changing the function call to `AASingle('Tightened')` results in a plan with a sample size of $n = 125$ with an acceptance number of $c = 3$ and a rejection number of $r = 4$.

ANSI/ASQ Z1.4 Double Sampling Plan

An ANSI/ASQ Z1.4 Double Sampling plan for the same lot size AQL and inspection level can be retrieved with the `AADouble()` function as shown below.

```
> library(AQLSchemes)
> planD<-AADouble('Normal')

What is the Inspection Level?

1: S-1
2: S-2
3: S-3
4: S-4
5: I
6: II
7: III

Selection: 6

What is the Lot Size?

 1: 2-8               2: 9-15              3: 16-25             4: 26-50
 5: 51-90             6: 91-150            7: 151-280           8: 281-500
 9: 501-1200         10: 1201-3200        11: 3201-10,000      12: 10,001-35,000
13: 35,001-150,000   14: 150,001-500,000  15: 500,001 and over

Selection: 10
```

ANSI/ASQ Z1.4 Double Sampling Plan

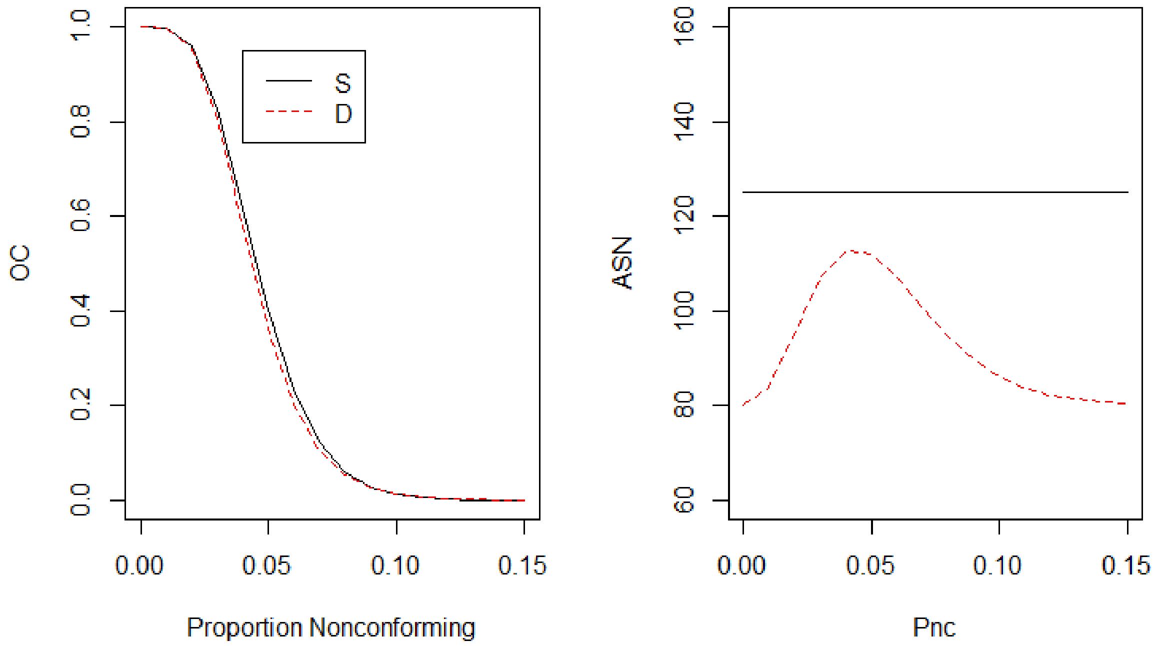
```
What is the AQL in percent nonconforming per 100 items?

 1: 0.010   2: 0.015   3: 0.025   4: 0.040   5: 0.065   6: 0.10    7: 0.15    8: 0.25
 9: 0.40   10: 0.65   11: 1.0    12: 1.5    13: 2.5    14: 4.0    15: 6.5    16: 10
17: 15     18: 25     19: 40     20: 65     21: 100    22: 150    23: 250    24: 400
25: 650    26: 1000

Selection: 12
> planD
        n c r
first  80 2 5
second 80 6 7
```

Comparing OC and ASN for Single and Double Plans

The R code below compares the OC and ASN curves for Single and Double Sampling Plans

```r
library(AQLSchemes)
par(mfcol=c(1,2))
Pnc<-seq(0,.15,.01)
SOCASN<-OCASNZ4S(planS,Pnc)
DOCASN<-OCASNZ4D(planD,Pnc)
OCS<-SOCASN$OC
OCD<-DOCASN$OC
ASND<-DOCASN$ASN
# plot OC Curves
plot(Pnc,OCS,type='l', xlab='Proportion Nonconforming', ylab='OC', lty=1)
lines(Pnc,OCD, type='l', lty=2,col=2)
legend(.04,.95,c("S","D"),lty=c(1,2),col=c(1,2))
# ASN Curves
plot(Pnc,ASND,type='l',ylab='ASN',lty=2,col=2,ylim=c(60,160))
lines(Pnc,rep(125,16),lty=1)
par(mfcol=c(1,2))
```

Comparing OC and ASN for Single and Double Plans



ANSI/ASQ Z1.4 Attribute Sampling Scheme

(Stephens and Larson, 1967) investigated the properties of MIL-STD-105E, which is also relevant to ANSI/ASQ Z1.4. When ignoring the reduced plan (the use of which requires authority approval), the sampling scheme again can be viewed as a two-state Markov chain with the two states being normal and tightened inspection.

The probability of accepting by this scheme is given by:

$$Pr(accept) = \frac{aP_N + bP_T}{a+b}$$

$$a = \frac{2 - P_N^4}{(1-P_N)(1-P_N^4)}$$

$$b = \frac{1 - P_T^5}{(1-P_T)P_T^5}.$$

$$ASN = \frac{an_N + bn_T}{a+b}.$$

ANSI/ASQ Z1.4 Attribute Sampling Scheme

The R code below creates a graph of the normal, tightened and scheme OC when $n_N = 50, c_N = 1$, and $n_T = 80, c_T = 1$

```
# Comparison of Normal, Tightened, and MIL-STD-105E OC Curves
library(AcceptanceSampling)
par(mfcol=c(1,2))
pd<-seq(0,.1,.001)
PN=pbinom(1,size=50,prob=seq(0,.1,.001),lower.tail=TRUE)
PT=pbinom(0,size=80,prob=seq(0,.1,.001),lower.tail=TRUE)
a=(2-PN^4)/((1.0000000000001-PN)*(1.0000000000001-PN^4))
b=(1.0000000000001-PT^5)/((1.0000000000001-PT)*(PT^5))
PS<-(a*PN+b*PT)/(a+b)
plot(pd,PS,type='l',lty=1,xlim=c(0,.1),xlab='Probability of nonconforming',ylab="OC")
lines(pd,PN,type='l',lty=2)
lines(pd,PT,type='l',lty=3)
lines(c(.05,.06),c(.9,.9),lty=1)
lines(c(.05,.06),c(.8,.8),lty=2)
lines(c(.05,.06),c(.7,.7),lty=3)
text(.08,.9,'Scheme')
text(.08,.8,'Normal')
text(.08,.7,'Tightened')
# ASN for Scheme
ASN<-(a/(a+b))*50+(b/(a+b))*80
pd<-seq(0,.1,.001)
plot(pd,ASN,type='l',ylim=c(5,100),xlim=c(0,.025),xlab="Probability of nonconforming",
     ylab="Scheme ASN")
par(mfcol=c(1,1))
```