Time Weighted Control Charts in Phase II

November 10, 2020

Outline

Phase II

In Phase II, the information obtained in Phase I (OCAP, process μ, σ are used to determine control chart limits for monitoring real time data. When an out of control signal is encountered, the OCAP is used to determine what adjustment is needed to bring the process back into control.

Phase II

In order to use control charts effectively to remove assignable cause variability in Phase II monitoring, the Automotive Industry Action Group recommends the following preparatory steps be taken.

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- Define the process
- Determine the characteristics to be charted
- Define the measurement system

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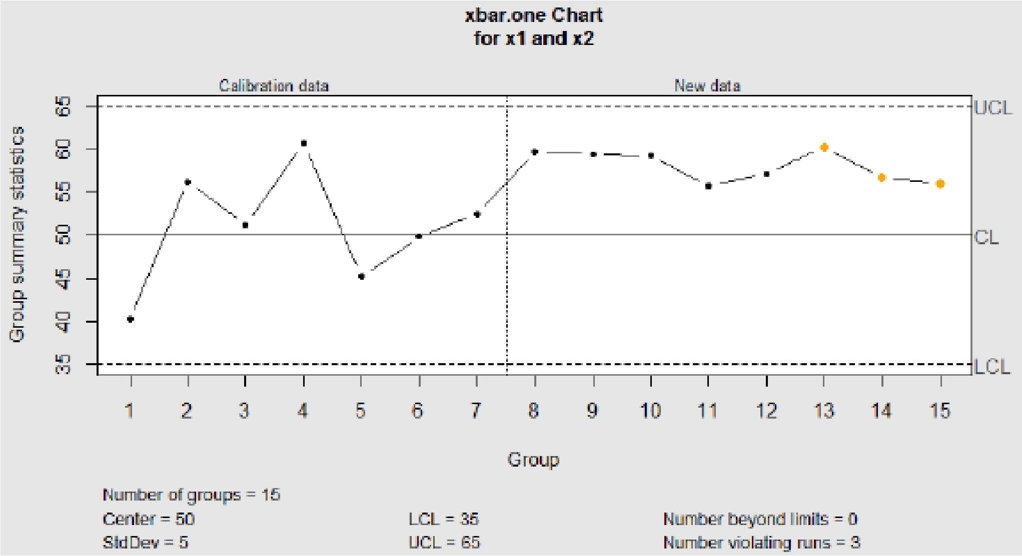
- Time Weighted Control Charts Plot Cumulative Sums of Past Observations and are More Sensitive to Small Shifts in the Mean
- Average Time to Signal = ARL×(time between Subgroup Measurements)
- ATS can be reduced by taking individual observation more frequently

Comparison of Shewhart Individuals Chart and Time Weighted Chart

```
library(qcc)
# random data from normal N(50,5) followed by N(56.6, 5)
set.seed(109)
x1<-rnorm(7,50,5)
set.seed(115)
x2<-rnorm(8,56.6,5)
# individuals chart assuming mu=50, sigma=5
library(qcc)
qcc(x1, type="xbar.one",center=50,std.dev=5,newdata=x2)
```



Comparison of Shewhart Individuals Chart and Time Weighted Chart

$$C_i = \sum_{k=1}^{i}(X_k - \mu),$$

Table 6.1 Cumulative Sums of Deviations from the Mean

Observation	Value	Deviation from μ=50	C_i
1	40.208	-9.792	-9.792
2	56.211	6.211	-3.581
3	51.236	1.236	-2.345
4	60.686	10.686	8.341
5	45.230	-4.770	3.571
6	49.849	-0.151	3.420
7	52.491	2.491	5.912
8	59.762	9.762	15.674
9	59.462	9.462	25.135
10	59.302	9.302	34.437
11	55.679	5.679	40.117
12	57.155	7.155	47.272
13	60.219	10.219	57.491
14	56.770	6.771	64.261
15	55.949	5.948	70.210

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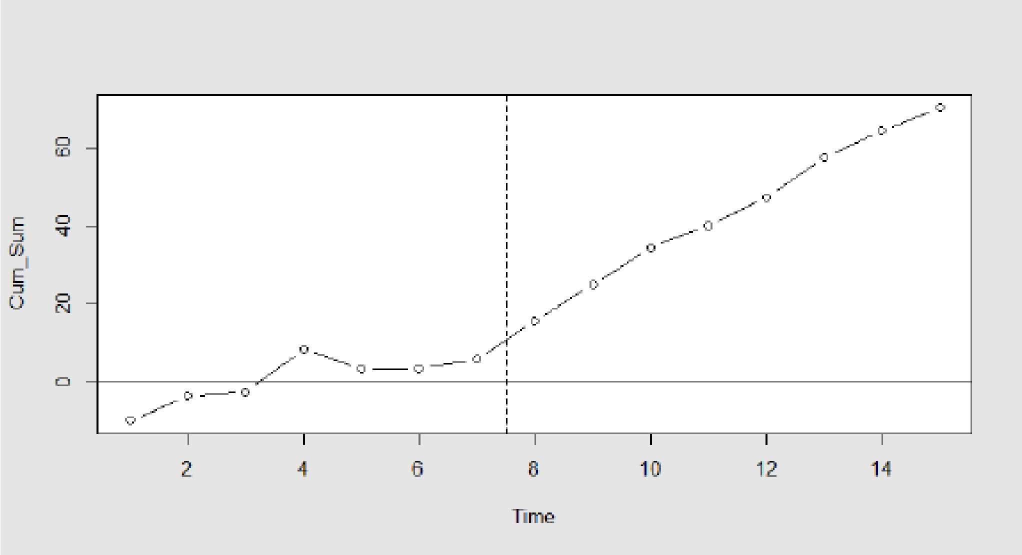


Although the mean shift is apparent there are no control limits

The Tabular Cusum Chart

$$C_i^+ = max[0, y_i - k + C_{i-1}^+]$$

$$C_i^- = max[0, -k - y_i + C_{i-1}^-]$$

The constant $k = 1/2$ is chosen to detect a 1σ shift in the mean, and control limits $\pm h$ with $h = 5$ to make $ARL_0 = 465$.

The Tabular Cusum Chart

$$C_i^+ = max[0, y_i - k + C_{i-1}^+]$$

$$C_i^- = max[0, -k - y_i + C_{i-1}^-]$$

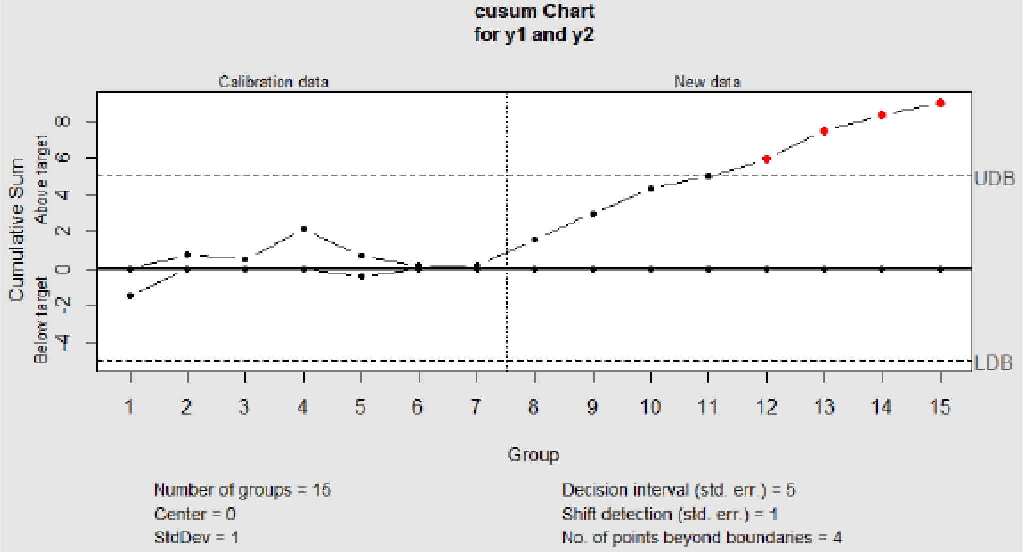
Table 6.2 Standardized Tabular Cusums

Individual value	x_i	y_i	C_i^+	$-C_i^-$
1	40.208	-1.958	0.000	-1.458
2	56.211	1.242	0.742	0.000
3	51.236	0.247	0.489	0.000
4	60.686	2.137	2.126	0.000
5	45.230	-0.954	0.673	-0.453
6	49.849	-0.0302	0.142	0.000
7	52.491	0.498	0.141	0.000
8	59.762	1.952	1.593	0.000
9	59.462	1.892	2.985	0.000
10	59.302	1.860	4.346	0.000
11	55.679	1.136	4.982	0.000
12	57.155	1.431	5.913	0.000
13	60.219	2.043	7.456	0.000
14	56.770	1.354	8.311	0.000
15	55.949	1.190	9.000	0.000

The Tabular Cusum Chart

The function `cusum()` in the `qcc` package plots C_i^+ on the upper half and $-C_i^-$ on the lower side of the graph

```
# standardized Cusum chart assuming mu=50, sigma=5
library(qcc)
y1<-(x1-50)/5
y2<-(x2-50)/5
cusum(y1,center=0,std.dev=1,se.shift=1,decision.interval=5,newdata=y2)
```



The Tabular Cusum Chart

When C_i^+ exceeds h, to give an out-of-control signal, an estimate of the current value of the process mean is given by Equation (6.3) .

$$\hat{\mu_c} = \mu + \sigma k + \frac{\sigma C_i^+}{N+}, \tag{6.3}$$

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For example, the Cusum chart in Figure 6.3 first exceeds $h = 5$ when $C_i^+ = 5.913$, and at that point there are $N^+ = 11$ consecutive positive values for C_i^+. So the estimate of the process mean at that point is:

$$\hat{\mu_c} = 50 + 5(.5) + \frac{(5)(5.913)}{11} = 55.19.$$

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When $-C_i^-$ is less than $-h$, to give an out-of-control signal, an estimate of the current mean of the process is given by:

$$\hat{\mu_c} = \mu - \sigma k - \frac{\sigma C_i^-}{N^+}, \tag{6.4}$$

Headstart or Fast Initial Response FIR

When the process is stopped due to an out-of-control signal, and an adjustment is made, the FIR allows faster detection of another out-of-control signal due to misadjustment

$$C_0^+ = h/2, \text{ and } C_0^- = -h/2$$

$$h = 5 \text{ so } C_0^+ = 2.5 \text{ and } C_0^- = -2.5$$

Headstart or Fast Initial Response FIR

Table 6.3 Standardized Tabular Cusums with FIR of $h/2$

Individual value	x_i	y_i	C_i^+	$-C_i^-$
1	40.208	-1.958	0.042	-3.958
2	56.211	1.242	0.784	-2.216
3	51.236	0.247	0.531	-1.469
4	60.686	2.137	2.168	0.000

ARL for Shewhart and Cusum for Phase II

```
#ARL individuals chart with 3 sigma limits
mu<-c(0.0,.5,1.0,2.0,3.0,4.0,5.0)
OC3=c(pnorm(3-mu[1])-pnorm(-3-mu[1]),pnorm(3-mu[2])-pnorm(-3-mu[2]),
      pnorm(3-mu[3])-pnorm(-3-mu[3]),pnorm(3-mu[4])-pnorm(-3-mu[4]),
      pnorm(3-mu[5])-pnorm(-3-mu[5]),pnorm(3-mu[6])-pnorm(-3-mu[6]),
      pnorm(3-mu[7])-pnorm(-3-mu[7]))
ARL3=1/(1-OC3)
```

```
#ARL for Cusum with and without the headstart feature
library(spc)
ARLC<-sapply(mu,k=.5,h=5,sided="two",xcusum.arl)
ARLChs<-sapply(mu,k=.5,h=5,hs=2.5,sided="two",xcusum.arl)
round(cbind(mu,ARL3,ARLC,ARLChs),digits=2)
```

ARL for Shewhart and Cusum for Phase II

Table 6.4 ARL for Shewhart and Cusum Control Charts

Shift in Mean	Shewhart	Cusum Chart	Cusum with $h/2$ headstart
Multiples of σ	Individuals Chart	$k = 1/2, h = 5$	$k = 1/2, h = 5$
0.0	370.40	465.44	430.39
0.5	155.22	38.00	28.67
1.0	43.89	10.38	6.35
2.0	6.30	4.01	2.36
3.0	2.00	2.57	1.54
4.0	1.19	2.01	1.16
5.0	1.02	1.69	1.02

EWMA - Weighted Cumulative Sums

For $0 < \lambda < 1$, weights $\lambda(1-\lambda)^m$ decrease exponentially

$$
\begin{aligned}
z_1 &= \lambda x_1 + (1-\lambda)\mu \\
z_2 &= \lambda x_2 + (1-\lambda)z_1 \\
&\vdots \\
z_n &= \lambda x_n + (1-\lambda)z_{n-1}
\end{aligned}
$$

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$$z_n = \lambda x_n + \lambda(1-\lambda)x_{n-1} + \lambda(1-\lambda)^2 x_{n-2} + ... + \lambda(1-\lambda)^{n-1}x_1 + (1-\lambda)^n\mu,$$

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$$z_n = \lambda x_n + \lambda(1-\lambda)x_{n-1} + \lambda(1-\lambda)^2 x_{n-2} + \ldots + \lambda(1-\lambda)^{n-1} x_1 + (1-\lambda)^n \mu,$$

$$z_n = \lambda x_n + (1-\lambda) z_{n-1} = \lambda \sum_{i=0}^{n-1} (1-\lambda)^i x_{n-i} + (1-\lambda)^n \mu, \qquad (6.5)$$

EWMA - Weighted Cumulative Sums

For $\lambda > .9$, EWMA $\equiv$ Shewhart individuals; for $\lambda < .1$, EWMA $\equiv$ Cusum

Table 6.5 EWMA Calculations $\lambda = 0.2$ Phase II Data from Table 6.2

Sample Number i	Observation x_i	EWMA $z_i = (0.2)x_i + (0.8)Z_{i-1}$
1	40.208	48.042
2	56.211	49.675
3	51.236	49.988
4	60.686	52.127
5	45.230	50.748
6	49.849	50.568
7	59.491	50.953
8	59.762	52.715
9	59.462	54.064
10	59.302	55.112

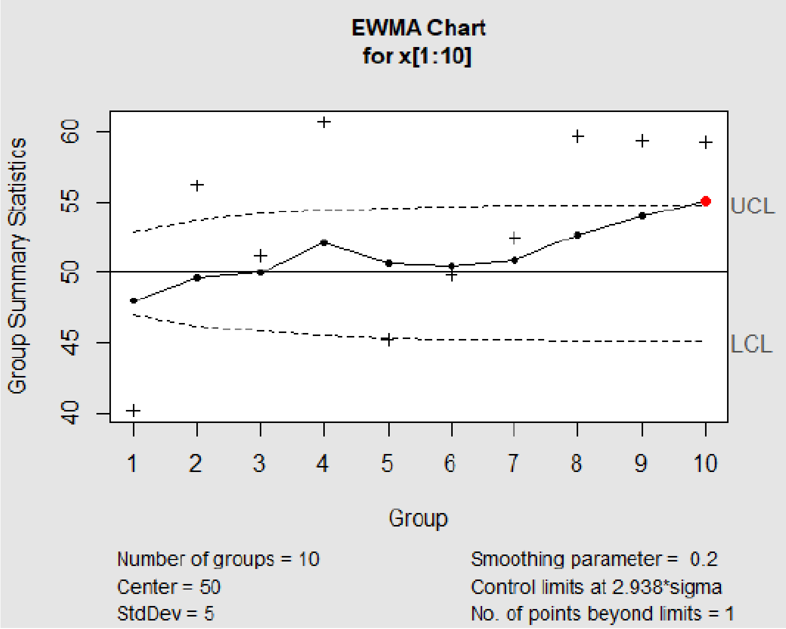
Control Limits for EWMA

$$UCL = \mu + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \tag{6.6}$$

$$LCL = \mu - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}, \tag{6.7}$$

Producing an EWMA Chart with R

```
# random data from normal N(50,5) followed by N(56.6, 5)
library(qcc)
set.seed(109)
x1<-rnorm(7,50,5)
set.seed(115)
x2<-rnorm(8,56.6,5)
x<-c(x1,x2)
# standardized ewma chart assuming mu=50, sigma=5
E1<-ewma(x[1:10],center=50,std.dev=5,lambda=.2,nsigmas=2.938)
```



EWMA as a Forecast

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Knowing this can help in deciding how to adjust the process to be back on target before the next observation, after an out-of-control signal.

ARL for an EWMA

The `xewma.arl()` function in the R package `spc` calculates the ARL for an EWMA Chart

```
library(spc)
xewma.arl(mu=0,l=.2,c=2.938,sided="two")
[1] 465.4878
```

Comparing ARL's for Different Charts

Table 6.6 ARL for Shewhart and Cusum Control Charts and EWMA Charts

Shift in Mean	Cusum Chart	EWMA with	Shewhart	EWMA with
Multiples of σ	$k = 1/2, h = 5$	$\lambda = 0.2$ and $L = 2.938$	Indiv. Chart	$\lambda = 0.4$ and $L = 2.9589$
0.0	465.44	465.48	370.40	370.37
0.5	38.00	40.36	155.22	58.45
1.0	10.38	10.36	43.89	12.71
2.0	4.01	3.71	6.30	3.35
3.0	2.57	2.36	2.00	1.95
4.0	2.01	1.85	1.19	1.39
5.0	1.69	1.46	1.02	1.10

EWMA with FIR Feature

Modified Control Limits

$$\pm L\sigma\left[\left(1-(1-f)^{1+a(t-1)}\right)\sqrt{\frac{\lambda}{2-\lambda}\,(1-(1-\lambda)^{2i}]}\right] \tag{6.8}$$

$$f = 0.5$$

$$a = [-2/\log(1-f)-1]/19$$

The Need to Detect Changes in σ during Phase II Monitoring

When the capability index (PCR) is equal to 1.00, a 31% to 32% increase in the process standard deviation will produce as much process fallout (or nonconforming output) as a 1σ shift in the process mean. For that reason, it is important to monitor the process standard deviation as well as the mean during Phase II.

Hawkins(1981)

If $x_i \sim$N(μ,σ^2),

then $\frac{\sqrt{|y_i|}-.822}{.349} \sim$N(0,1),

where $y_i = (x_i - \mu)/\sigma$.

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Changes in the standard deviation of x_i result in changes in the mean of v_i

Hawkins(1981)

If y_i follows a normal distribution with mean zero, and its standard deviation has increased from 1 to γ, the expected value of $\sqrt{|y_i|}$ can be found as shown in Equation (6.10).

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$$E[\sqrt{|y_i|}] = \frac{1}{\gamma\sqrt{2\pi}} \int_{\infty}^{\infty} \sqrt{|y_i|} \exp\left(\frac{-y_i^2}{2\gamma^2}\right) dy_i, \tag{6.10}$$

Using numerical integration, the expected values of $\sqrt{|y_i|}$ and v_i for four possible values of γ were found and are shown in Table 6.7.

Table 6.7 Expected value of $\sqrt{|y_i|}$ and v_i as function of γ

% Change in σ	γ	$E[\sqrt{|y_i|}]$	$E(v_i)$
-20%	0.8	0.735379	-0.24860
0%	1.0	0.822179	0.00000
32%	1.32	0.944612	0.35066
50%	1.5	1.006960	0.52923

Phase II Monitoring of μ and σ

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- This is analogous to the case when monitoring with subgrouped data, and for that reason $\overline{X}$ and R charts are kept together.
- Hawkins(1993) recommends keeping Cusum Chart of y_i together with Cusum Chart of v_i when monitoring individual values in Phase II.

Hawkins(1981) Recommended Cusum Chart for Monitoring v_i

$$C_i^+ = max[0, v_i - k + C_{i-1}^+] \tag{6.11}$$

$$C_i^- = max[0, -k - v_i + C_{i-1}^-], \tag{6.12}$$

where, $k = .25$ and the decision limit $h = 6$. He showed that if the process standard deviation increases, the Cusum chart for y_i used for monitoring the process mean (shown in column 1 of Table 6.8) may briefly cross its control or decision limit (h=5), but that the Cusum for v_i, described above, will cross and stay above its control or decision limit. Therefore, keeping the two charts together will help to distinguish between changes in the process mean and changes in the process standard deviation.

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This Cusum Chart has an $ARL_0 = 250.805$ and an ARL for detecting a 32% increase in the standard deviation is 33.51

An EWMA Chart for Monitoring v_i

EWMA charts can also be used to monitor changes in individual values like v_i. The EMMA at time point i is defined as:

$$z_i = \lambda v_i + (1 - \lambda) z_{i-1} \tag{6.13}$$

where $\lambda = 0.05$, and the control limits for the EWMA chart at time point i are given by:

$$LCL = -L\sqrt{\frac{\lambda}{2-\lambda}\left[1 - (1-\lambda)^{2i}\right]} \tag{6.14}$$

$$UCL = +L\sqrt{\frac{\lambda}{2-\lambda}\left[1 - (1-\lambda)^{2i}\right]} \tag{6.15}$$

An EWMA Chart for Monitoring v_i

The `xewma.crit` function in the R package `spc` can be used to determine the value of the multiplier L used in the formula for the control limits so that the ARL_0 of the chart will match that of (Hawkins, 1981)'s recommended Cusum chart for monitoring v_i. It is illustrated in the R code shown below where it was used to find the multiplier $L = 2.31934$ needed to produce an $\text{ARL}_0 = 250.805$ matching the Cusum chart with $k = .25$ and $h = 6$.

```
library(spc)
xewma.crit(l=.05,L0=250.8,mu0=0,
sided="two")
```

Comparing ARL of Charts for Monitoring σ with Individual Values

Table 6.8 ARL for Detecting Increases in σ by Monitoring v_i

% Increase in	Cusum Chart	EWMA	Shewhart Individuals Chart
σ	$k = .25, h = 6$	$\lambda = 0.05$ and $L = 2.24797$	$\pm 2.88\sigma$ Limits
0%	250.805	250.800	250.824
32%	33.51	33.37	157.58
50%	19.39	19.98	102.94

Example 1 Cusum Chart to Monitor Variability

As an example of a Cusum chart to monitor process variability with individual values, consider the data shown in the R code below. This data comes from (Summers 2000) and represents the diameter of spacer holes in surgical tables.

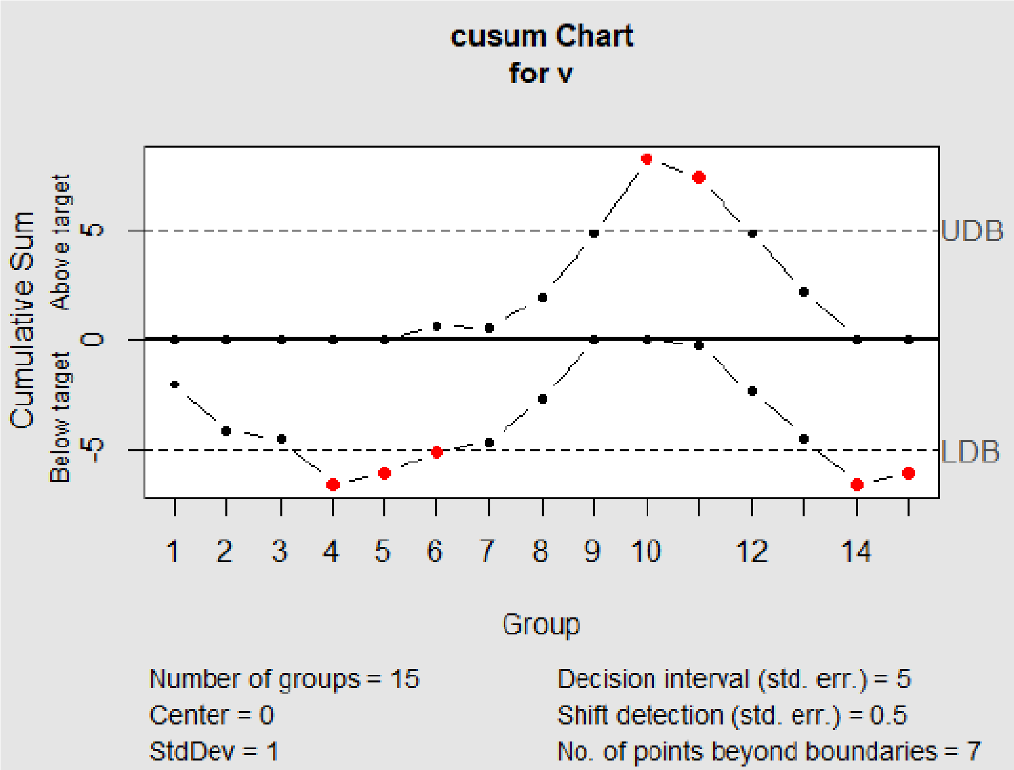
There is no known mean or standard deviation for this data, but there are known specification limits of 0.25 ± 0.01. If the mean was $\mu = 0.25$ and the standard deviation was $\sigma = 0.0025$, then $C_p = C_{pk} = 1.33$. A shift of the mean away from $\mu = 0.25$ by more than 0.0025 in either direction would result in $C_{pk} < 1.0$, and an increase in the standard deviation by more than 50% would also result in $C_p < 1.0$. Therefore, although the process mean and standard deviation are unknown, Phase II monitoring of changes in the process mean and standard deviation from the values known to result in $C_p = C_{pk} = 1.33$ could begin, and there is no need for a Phase I study to estimate μ and σ.

Example 1 Cusum Chart to Monitor Variability

In the code below, **mu** and **sigma** Are assigned the values that would result in $C_{pk} = 1.33$

```
diameter<-c(.25,.25,.251,.25,.252,.253,.252,.255,.259,.261,.249,.250,.250,.250,.252)

mu<-.25

sigma<-.0025

y<-(diameter-mu)/sigma

v<-(sqrt(abs(y))-.822179)/.3491508

library(qcc)

cusum(v,center=0,std.dev=1,decision.interval=6,se.shift=.5)
```

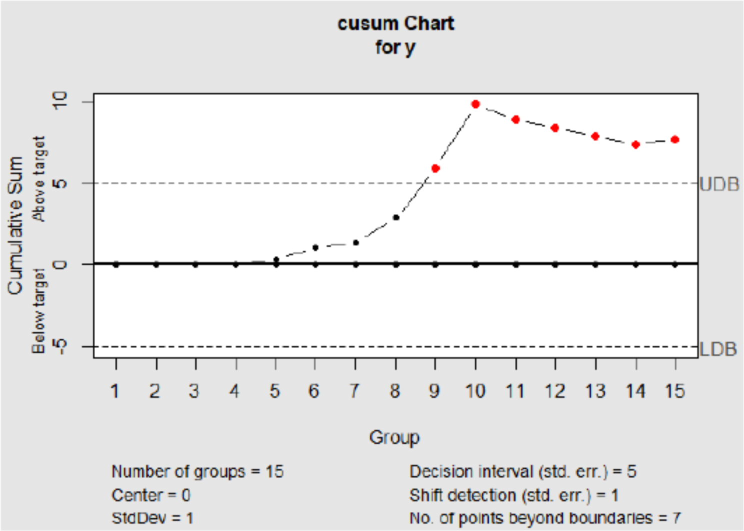
Example 1 Cusum Chart to Monitor Variability



It seems strange that the variability would increase and immediately return to a in-control state.

Example 1 Cusum Chart of Standardized Values y_i

```
library(qcc)
cusum(y,center=0,std.dev=1,decision.interval=5,se.shift=1)
```

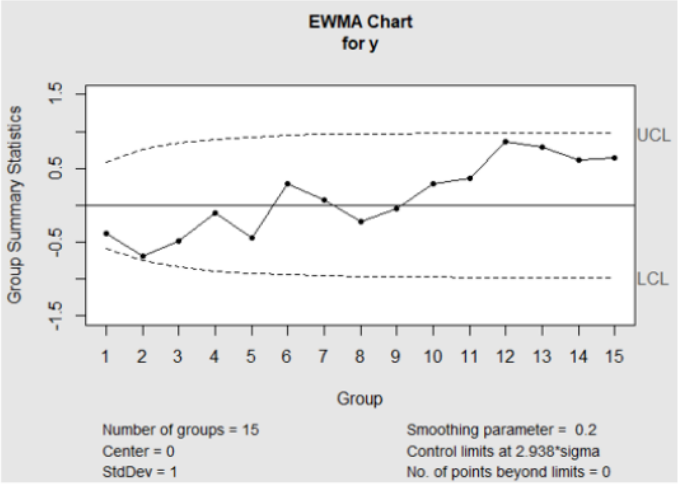


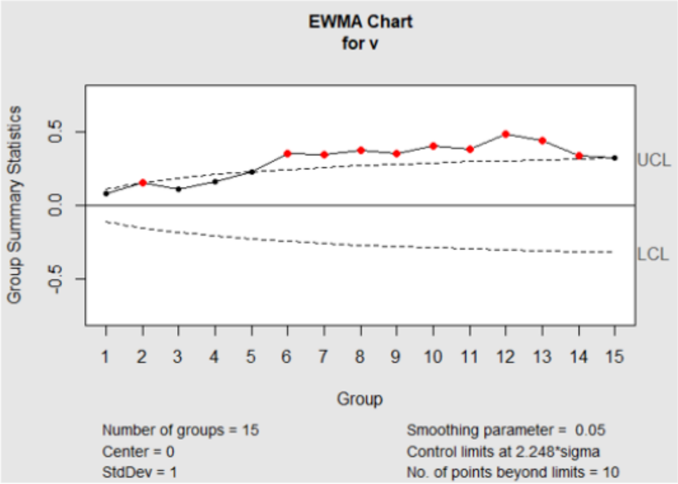
This appears to be due to the fact that the mean changed at the the same time as the temporary change in the standard deviation.

Example 2 Change in σ but no change in μ

```
set.seed(29)
# random data from normal N(50, 7.5)
x<-rnorm(15,50,7.5)
# standardize assuming sigma=5, mu=50
y<-(x-50)/5
# calculate v
v<-(sqrt(abs(y))-.822179)/.3491508
library(qcc)
EWMA <- ewma(y, center=0, std.dev=1,
lambda=.2, nsigmas=2.938, plot = FALSE)
EWMA$statistics <- rep(NA,
length(EWMA$statistics))
plot(EWMA,ylim=c(-1.5,1.5))
EWMA <- ewma(v, center=0, std.dev=1,
lambda=.05, nsigmas=2.248,
plot = FALSE)
EWMA$statistics <- rep(NA,
length(EWMA$statistics))
plot(EWMA, ylim=c(-.75,.75))
```

Example 2 Change in σ but no change in μ





Time Weighted Control Charts Using Phase I Estimates of μ and σ

When the the in-control process mean and standard deviation are unknown, the standard recommendation (see (Christensen et al., 2013)) is to use the estimates $\hat{\mu} = \overline{\overline{X}}$ and $\hat{\sigma} = \overline{R}/d_2$ from a Phase I study using $\overline{X} - R$ charts with subgroups of size $n = 4$ or 5, and $m = 25$ subgroups. But of course these quantities are random variables.

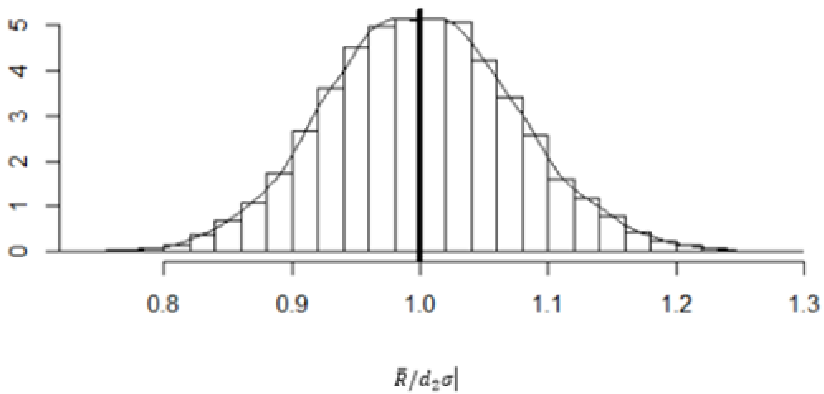


Figure 6.8 Simulated Distribution of $\overline{R}/d_2\sigma$

Time Weighted Control Charts Using Phase I Estimates of μ and σ

- 50% of the time $\hat{\sigma} = \overline{R}/d_2$ will be less than σ

Time Weighted Control Charts Using Phase I Estimates of μ and σ

- 50% of the time $\hat{\sigma} = \overline{R}/d_2$ will be less than σ
- 10% of the time $\hat{\sigma} = \overline{R}/d_2$ will be less than 90% of σ

Time Weighted Control Charts Using Phase I Estimates of μ and σ

- 50% of the time $\hat{\sigma} = \overline{R}/d_2$ will be less than σ
- 10% of the time $\hat{\sigma} = \overline{R}/d_2$ will be less than 90% of σ
- Width of control limits is a multiple of $\hat{\sigma} = \overline{R}/d_2$, and they will be too narrow 50% of the time

Time Weighted Control Charts Using Phase I Estimates of μ and σ

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- ARL_0 is decreased and chance of spurious out-of-control signals is increased

Time Weighted Control Charts Using Phase I Estimates of μ and σ

- 50% of the time $\hat{\sigma} = \overline{R}/d_2$ will be less than σ
- 10% of the time $\hat{\sigma} = \overline{R}/d_2$ will be less than 90% of σ
- Width of control limits is a multiple of $\hat{\sigma} = \overline{R}/d_2$, and they will be too narrow 50% of the time
- ARL_0 is decreased and chance of spurious out-of-control signals is increased
- To prevent tampering, the multiplier of $\hat{\sigma}$ in the formula for control limits should be increased.

Time Weighted Control Charts Using Phase I Estimates of μ and σ

- Gandy and Kvaloy(2013) recommended that a control chart be designed so that the ARL_0 achieve the desired value with a specified probability.

Time Weighted Control Charts Using Phase I Estimates of μ and σ

- Gandy and Kvaloy(2013) recommended that a control chart be designed so that the ARL_0 achieve the desired value with a specified probability.
- They propose a method based on bootstrap samples to find the 90th percentile of the control limit multiplier to guarantee the ARL_0 be at least the desired value 90% of the time.

Time Weighted Control Charts Using Phase I Estimates of μ and σ

The R package `spcadjust` (Gandy and Kvaloy, 2015) contains the function `SPCproperty()` that computes the adjusted multiplier for the control limits of the two-sided EWMA chart based on Gandy and Kvaloy's bootstrap method.

Time Weighted Control Charts Using Phase I Estimates of μ and σ

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- `SPCproperty()` can be used in place of `xewma.crit()` to find a control chart multiplier that will result in a desired ARL_0

Time Weighted Control Charts Using Phase I Estimates of μ and σ

The R package `spcadjust` (Gandy and Kvaloy, 2015) contains the function `SPCproperty()` that computes the adjusted multiplier for the control limits of the two-sided EWMA chart based on Gandy and Kvaloy's bootstrap method.

- `SPCproperty()` can be used in place of `xewma.crit()` to find a control chart multiplier that will result in a desired ARL_0
- Gandy and Kvaloy(2015) found this only increases ARL_1 slightly

Example-Time Weighted Control Charts Using Phase I Estimates of μ and σ

The process mean is $\mu = 50$, and the process standard deviation is $\sigma = 5$, but these are unknown. The R code below simulates 100 observations from a Phase I study

```r
# Simulate Phase I study with 100 observations
set.seed(99)
X <-  rnorm(100,50,5)
```

Example continued 1

The "SPCEWMA" class in the R package **spcadjust** specifies the parameters of the EWMA chart. The option `Delta=0` in the `SPCModelNormal` call below indicates that you want to find the adjusted multiplier that will guarantee the ARL_0 be greater than a desired value with a specified probability.

```
library(spcadjust)
chart <- new("SPCEWMA",model=SPCModelNormal(Delta=0),lambda=0.2);
xihat <- xiofdata(chart,X)
str(xihat)
```

Example continued 1

The "SPCEWMA" class in the R package **spcadjust** specifies the parameters of the EWMA chart. The option `Delta=0` in the `SPCModelNormal` call below indicates that you want to find the adjusted multiplier that will guarantee the ARL_0 be greater than a desired value with a specified probability.

```
library(spcadjust)
chart <- new("SPCEWMA",model=SPCModelNormal(Delta=0),lambda=0.2);
xihat <- xiofdata(chart,X)
str(xihat)
```

The **xiofdata** function computes the Phase I estimated parameters from the simulated data `X`. In this case $\hat{\mu} = \overline{X}$=`xihat$mu=49.5`, and $\hat{\sigma} = s$=`xihat$sd=4.5` were calculated from the simulated Phase I data.

Example continued 2

Next, the **SPCproperty** function in the **spcadjust** package is called to get the adjusted multiplier L for the EWMA chart. Actually this function computes $L \times \sqrt{\frac{\lambda}{2-\lambda}}$ where L is the adjusted multiplier. The option **target=465.48** specifies the desired value of ARL_0, and $\lambda = .2$ was specified in **"SPCEWMA"** class above. The function call is shown below.

```
library(spcadjust)
cal <- SPCproperty(data=X,nrep=1000,
                   property="calARL",chart=chart,params=list(target=465.48),quiet=TRUE)
cal
90 % CI: A threshold of +/- 1.128 gives an in-control ARL of at least 465.48.
Unadjusted result:  0.9795
Based on  1000 bootstrap repetitions.
```

$$L \times \sqrt{\frac{\lambda}{2-\lambda}} = 1.128 \text{ gives an in-control ARL of at least 465.48.}$$

Example continued 2

Next, the **SPCproperty** function in the **spcadjust** package is called to get the adjusted multiplier L for the EWMA chart. Actually this function computes $L \times \sqrt{\frac{\lambda}{2-\lambda}}$ where L is the adjusted multiplier. The option **target=465.48** specifies the desired value of ARL_0, and $\lambda = .2$ was specified in **"SPCEWMA"** class above. The function call is shown below.

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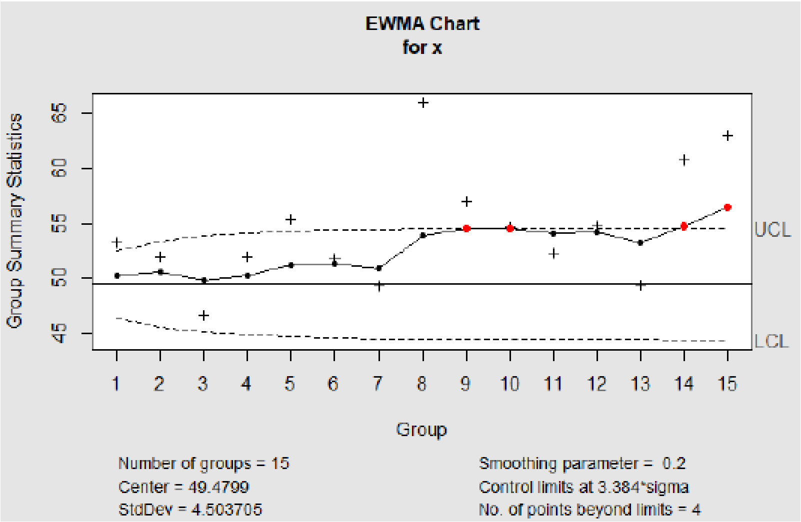
library(spcadjust)
cal <- SPCproperty(data=X,nrep=1000,
                   property="calARL",chart=chart,params=list(target=465.48),quiet=TRUE)
cal
90 % CI: A threshold of +/- 1.128 gives an in-control ARL of at least 465.48.
Unadjusted result:  0.9795
Based on  1000 bootstrap repetitions.
```

$L \times \sqrt{\frac{\lambda}{2-\lambda}}$=1.128 gives an in-control ARL of at least 465.48.

$$\text{Thus, } L = \frac{1.128}{\sqrt{\frac{.2}{2-.2}}} = 3.384$$

Example continued 3

```
# Simulate Phase II data with 1 sigma shift in the mean
set.seed(49)
x<-rnorm(15,55,5)
library(qcc)
ewma(x,center=xihat$mu,std.dev=xihat$sd,lambda=.2,nsigmas=3.384,plot=FALSE)
ex$statistics<-rep(NA,length(ex$statistics))
plot(ex,ylim=(c(40,60)))
```



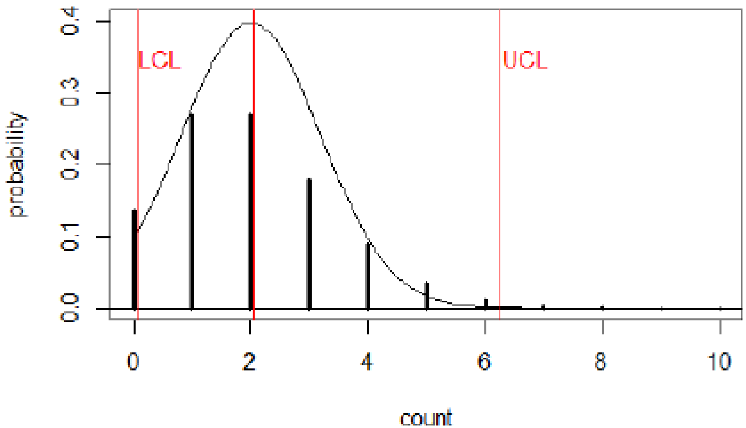
This detected a one σ shift in the mean at observation 9 with $ARL_0 > 465.48$

Time Weighted Control Charts for Phase II Monitoring of Attribute Data

Cusum charts can be used for counts of the number of nonconforming items per subgroup or the number of nonconformities per inspection unit in place of the Shewhart np chart or c chart. The Cusum charts will have a shorter ARL for detecting an increase or decrease in the average count and are more accurate. Therefore, they are very useful for processes where there are only counts rather than numerical measures to monitor.

Time Weighted Control Charts for Phase II Monitoring of Attribute Data

When λ is small normal approximation to Poisson used to obtain control limits for the c-chart is poor, and you cannot find an out-of-control signal for a reduction in non-conformities. The averaging from time-weighted charts is better.



Time Weighted Control Charts for Phase II Monitoring of Attribute Data

Lucas[68] developed a Cusum control scheme for counted data. Of the Cusums shown in Equation 6.16, C_i^+ is used for detecting an increase in D_i, the count of nonconforming items per subgroup (or the number of nonconformities per inspection unit). C_i^- is used for detecting a decrease in the count.

$$C_i^+ = max[0, D_i - k + C_{i-1}^+] \tag{6.16}$$

$$C_i^- = max[0, k - D_i + C_{i-1}^-]$$

He recommended the reference value k be determined by Equation 6.17, rounded to the nearest integer so that the cusum calculations only require integer arithmetic.

$$k = \frac{\mu_d - \mu_a}{\ln(\mu_d/\mu_a)} \tag{6.17}$$

Time Weighted Control Charts for Phase II Monitoring of Attribute Data

Lucas's Cusum control scheme for counted data

- The ARL is shorter than comparable c chart

Time Weighted Control Charts for Phase II Monitoring of Attribute Data

Lucas's Cusum control scheme for counted data

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Time Weighted Control Charts for Phase II Monitoring of Attribute Data

Lucas's Cusum control scheme for counted data

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- Lucas(1985) provided extensive tables of ARL indexed by H and k for detecting increase or decrease in mean count with or without FIR

Time Weighted Control Charts for Phase II Monitoring of Attribute Data

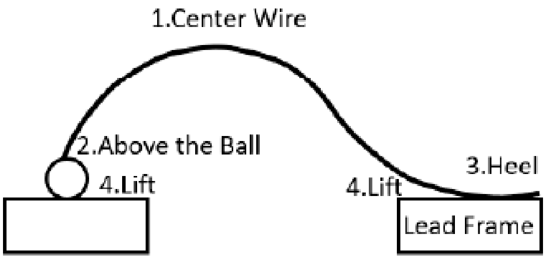
Lucas's Cusum control scheme for counted data

- The ARL is shorter than comparable c chart
- The Cusum is robust to the distribution assumption (i.e., Binomial or Poisson)
- Lucas(1985) provided extensive tables of ARL indexed by H and k for detecting increase or decrease in mean count with or without FIR
- An R function will be shown below to calculate ARL

Consider the application presented by White et. al. (1997)

"Motorola's Sensor Products Division is responsible for the production of various lines of pressure sensors and accelerometers. Each of these devices consists of a single silicon die mounted in a plastic package with multiple electrical leads. In one of the final assembly steps in the manufacturing process, the bond pads on the silicon chip are connected to the package leads with very thin (1-2 μ) gold wire in the process known as wire bonding. During this process, the wire is subjected to an electrical charge that welds it to its connections. The amount of energy used in the bond must be carefully controlled because too little energy will produce a bond that is weak and too much energy will severely weaken the wire above the bond and cause subsequent failure of the connection. To maintain control of this process, destructive testing is conducted at regular intervals by selecting a number of units and pulling on the wires until they break. The pull strength as well as the location of the break are recorded. Statistical control of the pull strength falls within the realm of traditional SPC techniques, as these are continuous variable measurements and roughly normal independent and identically distributed. The location of the break is known to engineers as the failure mode. This qualitative variable provides additional information about the state of control for the process ... Failure modes for the wire pull destructive testing procedure can take on four values (see Figure 6.11 taken from (White et al., 1997)) ... Failure at area 3 is known as a postbond heel break and is a cause of concern if the rate for this failure gets too large. Failure in this area is an indication of an overstressed wire caused by excessive bond energy."

Application presented by White et. al. (1997) 2



The number of post bond heel breaks (D_i) were observed in samples of 16 destructive wire pull tests. If the probability of a the failure mode being the post bond heel break is p, then the D_i follows an Binomial(16,p) distribution, and the mean or expected number would be $16p$. The acceptable mean rate was $\mu_a = 1.88$ (established as either a target or through a Phase I study). An unacceptably high mean rate was defined to be $\mu_d = 3.2$.

Application presented by White et. al. (1997) 3

If a c-chart was used to monitor the number of post bond heel breaks in Phase II, it would have a center line $\overline{c}$=1.88, with the upper control limit $UCL = \overline{c} + 3 \times \sqrt{\overline{c}} = 5.99$, and lower control limit = 0. The average run length (ARL) when there is no change in mean count can be calculated using the `ppois` function in R as:

$ARL_{\lambda=1.88}$ = 1/(1-`ppois(5,1.88)`)=79.28,

$ARL_{\lambda=3.2}$ = 1/(1-`ppois(5,3.2)`)=9.48.

Application presented by White et. al. (1997) 3

If Lucas's Cusum control scheme for counted data was used in Phase II instead of the c-chart, k would be calculated to be:

$$k = \frac{\mu_d - \mu_a}{\ln(\mu_d/\mu_a)} = \frac{3.2 - 1.88}{ln(3.2) - ln(1.88)} = 2.48, \tag{6.19}$$

which was rounded to the nearest integer $k = 2.0$.

With $k = 2.0$, the next step is to choose h. Larger values of h will result in longer ARL's both when $\lambda = \mu_a = 1.88$, and when $\lambda = \mu_d = 3.2$.

ARL function

The function `arl` in the R package `IAcsSPCR` (that contains the data and functions from this book) was used to calculate $\text{ARL}_{\lambda=1.88}$, and $\text{ARL}_{\lambda=3.2}$ for the Cusum control scheme for counted data with k held constant at 2.0, and various values for h. The function calls in the block of code below were used to calculate the entries in Table 6.9.

```
library(IAcsSPCR)
arl(h=6,k=2,lambda=1.88,shift=0)
arl(h=6,k=2,lambda=1.88,shift=.9627)
arl(h=8,k=2,lambda=1.88,shift=0)
arl(h=8,k=2,lambda=1.88,shift=.9627)
arl(h=10,k=2,lambda=1.88,shift=0)
arl(h=10,k=2,lambda=1.88,shift=.9627)
arl(h=12,k=2,lambda=1.88,shift=0)
arl(h=12,k=2,lambda=1.88,shift=.9627)
```

Application presented by White et. al. (1997) 4

When $h = 6$, and $k = 2$, for the case where λ remains constant at 1.88, the ARL is calculated with the function call `arl(h=6,k=2,lambda=1.88,shift=0)`.

When $h = 6$, and $k = 2$, for the case where λ has increased to 3.2, the ARL is calculated with the function call `arl(h=6,k=2,lambda=1.88,shift=.9627)` since $3.2 = \lambda = 1.88 + .9627 \times \sqrt{1.88}$, or an increase in the mean by 0.9627 standard deviations. The resulting ARLs for h = 6, 8, 10, and 12 are

Table 6.9 ARL for various values of h with $k = 2$, $\lambda_0 = 1.88$

Value of h	$ARL_{\lambda=1.88}$	$ARL_{\lambda=3.2}$
6	37.20	5.49
8	66.52	7.16
10	108.60	8.82
12	166.98	10.49

Application presented by White et. al. (1997) 5

TABLE 6.10: Calculations for Counted Data Cusum for Detecting an Increas with $k = 2$, $h = 10$

Sample No. i	D_i	$D_i - k$	C_i^+ with No FIR	C_i^+ with FIR $h/2$
1	3	1	1	6
2	1	-1	0	5
3	4	2	2	7
4	1	-1	1	6
5	3	1	2	7
6	1	-1	1	6
7	5	3	4	9
8	4	2	6	11
9	5	3	9	-
10	5	3	12	-

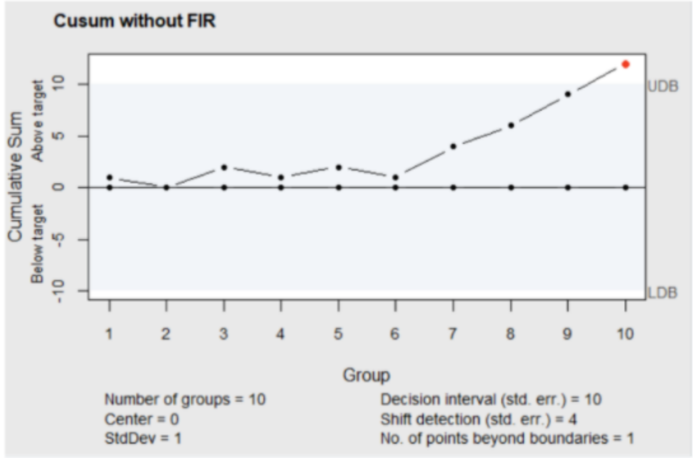
Application presented by White et. al. (1997) 6

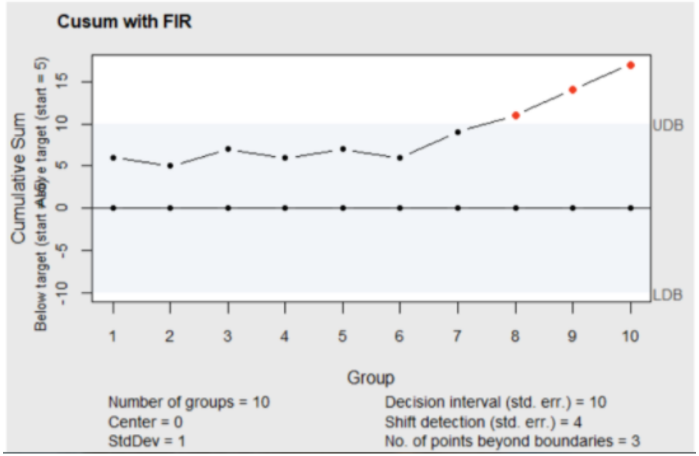
By specifying **center=0** and **std.dev=1**, in the calls to the **cusum()** function, the values of C_i^+ shown in Equation (6.16) without the FIR feature can be found in the named element **CUSUM$pos** of the object **CUSUM**, and the vector of values for C_i^+ with the FIR feature can be found in the named element **FIR$pos** of the object **FIR**. $C_0^+ = 0.0$ for the Cusum without FIR, and $C_0^+ = 5$ for the Cusum with FIR. The number of postbond heel breaks is stored in the vector **D**.

When monitoring to detect an improvement or reduction in the Poisson process mean, μ_d will be less than μ_a , and in that case the values the values of $-C_i^-$ the negatives of C_i^- shown in Equation (6.16) can be found in the element **CUSUM$neg** when the call to the **cusum()** function is

```
CUSUM<-cusum(D,center=0,std.dev=1,decision.interval=10,se.shift=-6)
```

Application presented by White et. al. (1997) 7





ARL Comparison of c-chart and Cusum for Attribute Data

Table 6.11 Comparison of ARL for c chart and Cusum for counted data ($k = 5$, $h = 10$) with target $lambda = 4$

λ	ARL for c chart	ARL for Cusum without FIR	ARL for Cusum with FIR
4	352.14	421.60	397.5
5	73.01	29.81	22.38
6	23.46	9.73	6.11
7	10.15	5.59	3.35
10	2.40	2.58	1.58

EWMA for Counted Data (Borror, Champ, and Rigdon 1998)

$$\text{EWMA } (z_i = \lambda x_i + (1-\lambda) z_{i-1},$$

$$UCL = \mu_0 + A\sqrt{\frac{\lambda \mu_0}{2-\lambda}}$$

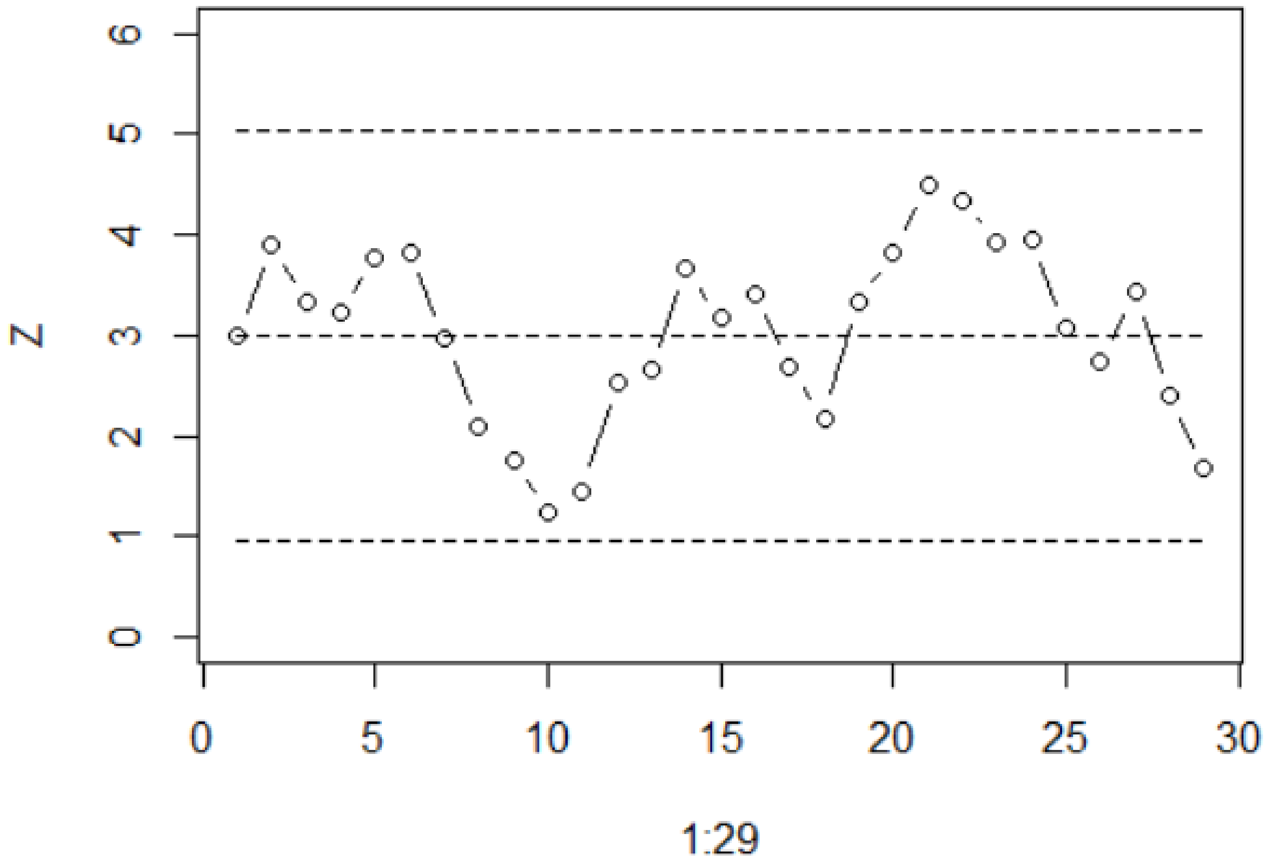
$$LCL = \mu_0 - A\sqrt{\frac{\lambda \mu_0}{2-\lambda}}$$

EWMA for Counted Data (Borror, Champ, and Rigdon 1998)

Example: Assuming μ_0=3, $\lambda = .3$, $A = 2.8$, and using the count data from Table 14.2 in Christensen et. al., the EWMA chart is made with the following R code.

```
#data from Table 14.1 Christensen
d<-c(3,6,2,3,5,4,1,0,1,0,2,5,3,6,2,4,1,1,6,5,6,4,3,4,1,2,5,0,0)
# initialize Z to zeros
Z<-c(rep(0,29))
lambda<-.3
mu<-3
A<-2.8
Z[1]<-lambda*d[1]+(1-lambda)*mu
for (i in 2:29) {
  Z[i]<-lambda*d[i]+(1-lambda)*Z[i-1]
}
UCL<-mu+A*sqrt((lambda*mu)/(2-lambda))
LCL<-mu-A*sqrt((lambda*mu)/(2-lambda))
plot(1:29,Z,type='b',ylim=c(0,6))
lines(1:29,rep(mu,29),type='l',lty=2)
lines(1:29,rep(UCL,29),type='l',lty=2)
lines(1:29,rep(LCL,29),type='l',lty=2)
```

EWMA for Counted Data (Borror, Champ, and Rigdon 1998)



ARL for Counted Data EWMA (Borror, Champ, and Rigdon 1998)

Assuming μ_0=4.0, $\lambda = .3$, $A = 2.8$, what is ARL_0 for a false positive signal.

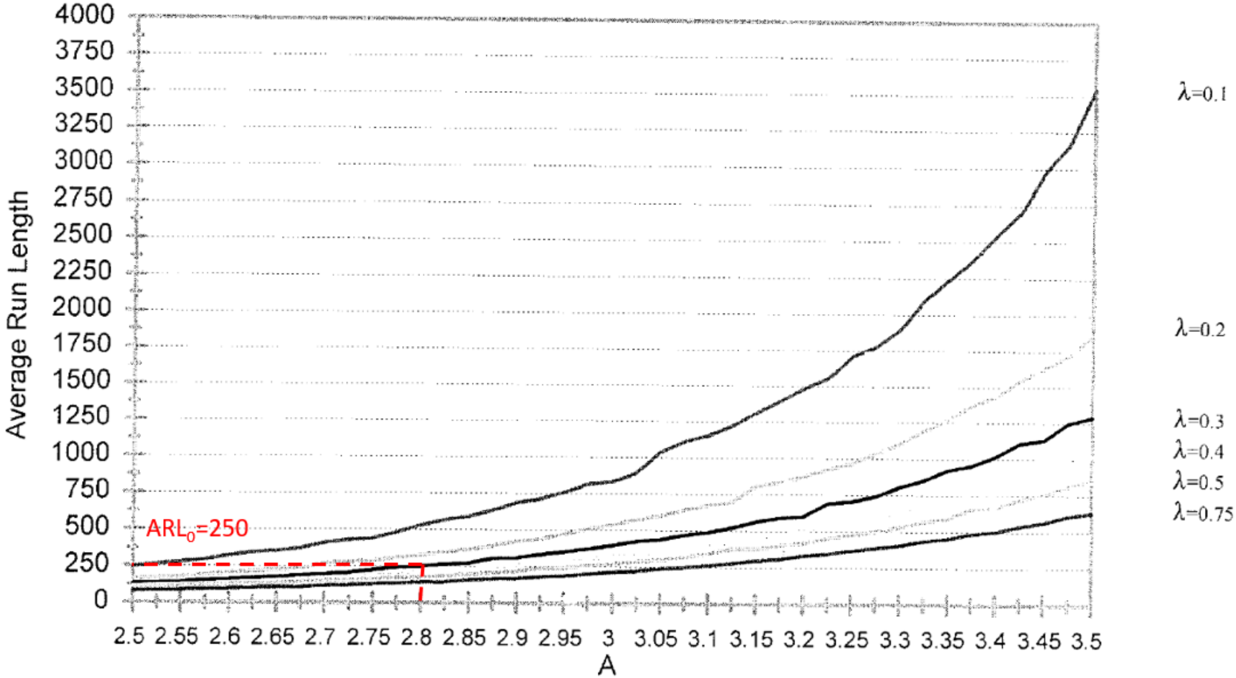


FIGURE 2. ARL's For Various Values of λ and A With an In-Control Mean of 4.0.

ARL for Counted Data EWMA (Borror, Champ, and Rigdon 1998)

μ_0=5.0 ARL_0 for a false positive signal.

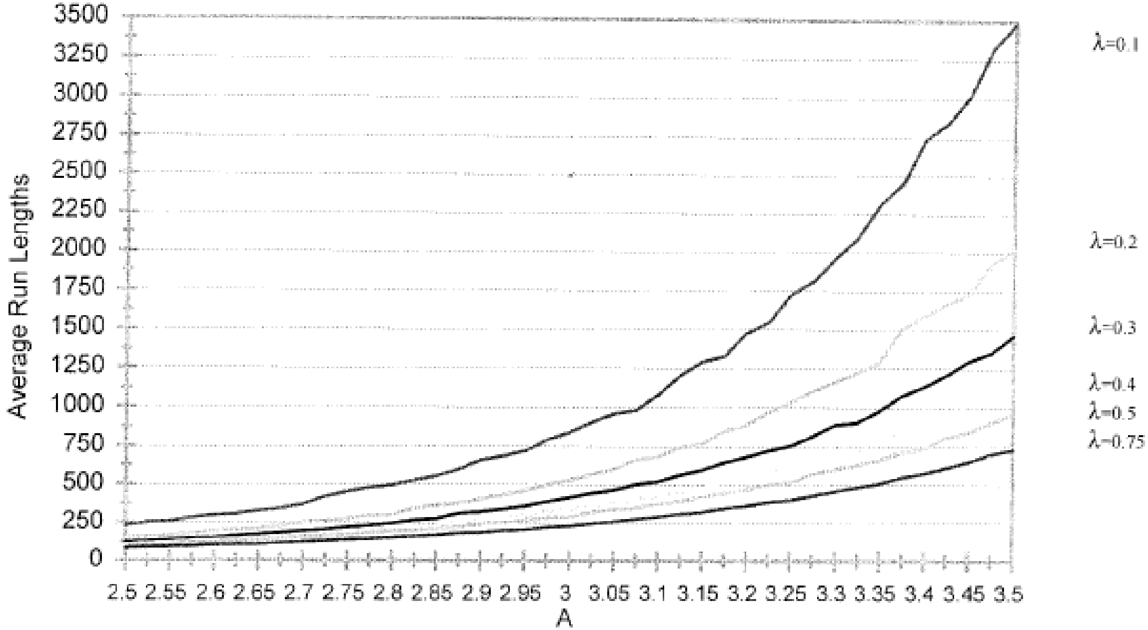


FIGURE 3. ARL's For various Values of λ and A With an In-Control Mean of 5.0.

ARL for Counted Data EWMA (Borror, Champ, and Rigdon 1998)

μ_0=6.0 ARL_0 for a false positive signal.

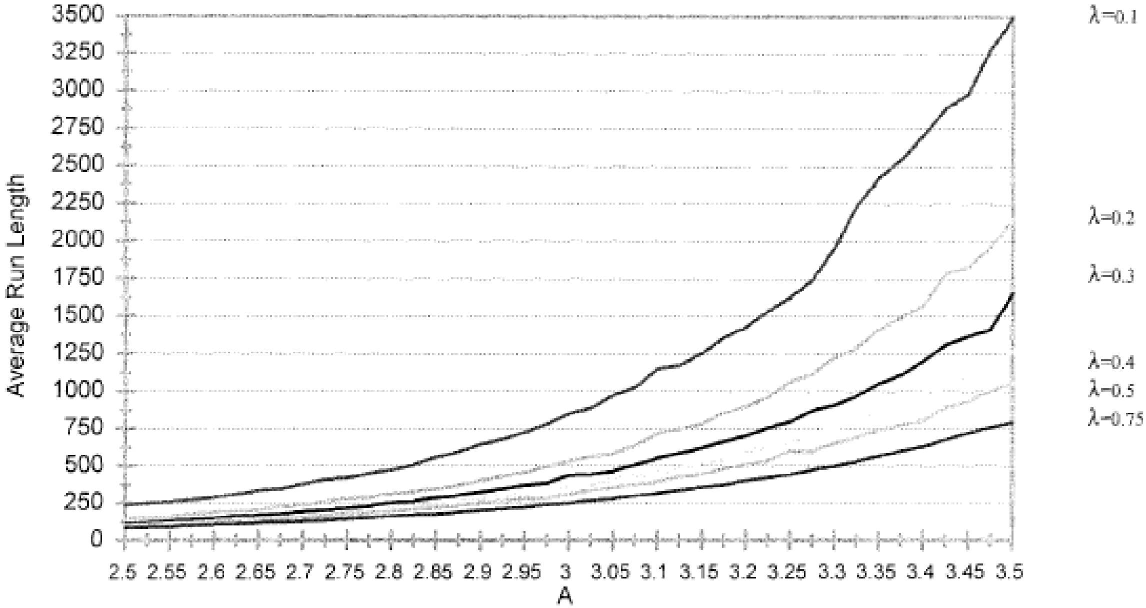


FIGURE 4. ARL's For various Values of λ and A With an In-Control Mean of 6.0.

ARL for Counted Data EWMA (Borror, Champ, and Rigdon 1998)

μ_0=7.0 ARL_0 for a false positive signal.

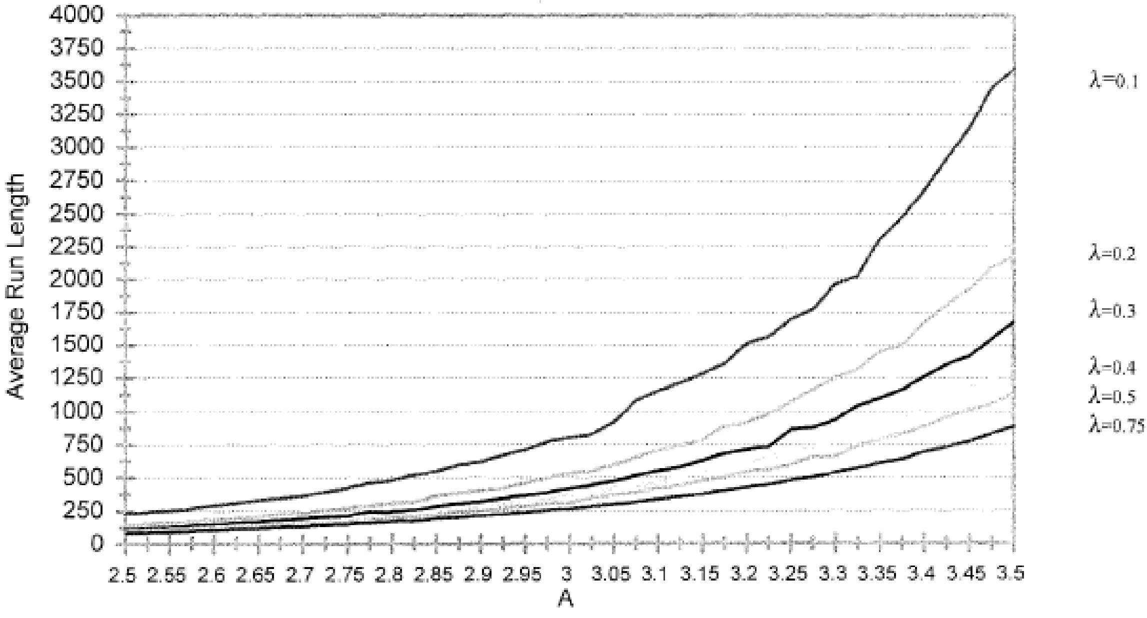


FIGURE 5. ARL's For various Values of λ and A With an In-Control Mean of 7.0.

ARL for Counted Data EWMA (Borror, Champ, and Rigdon 1998)

μ_0=8.0 ARL_0 for a false positive signal.

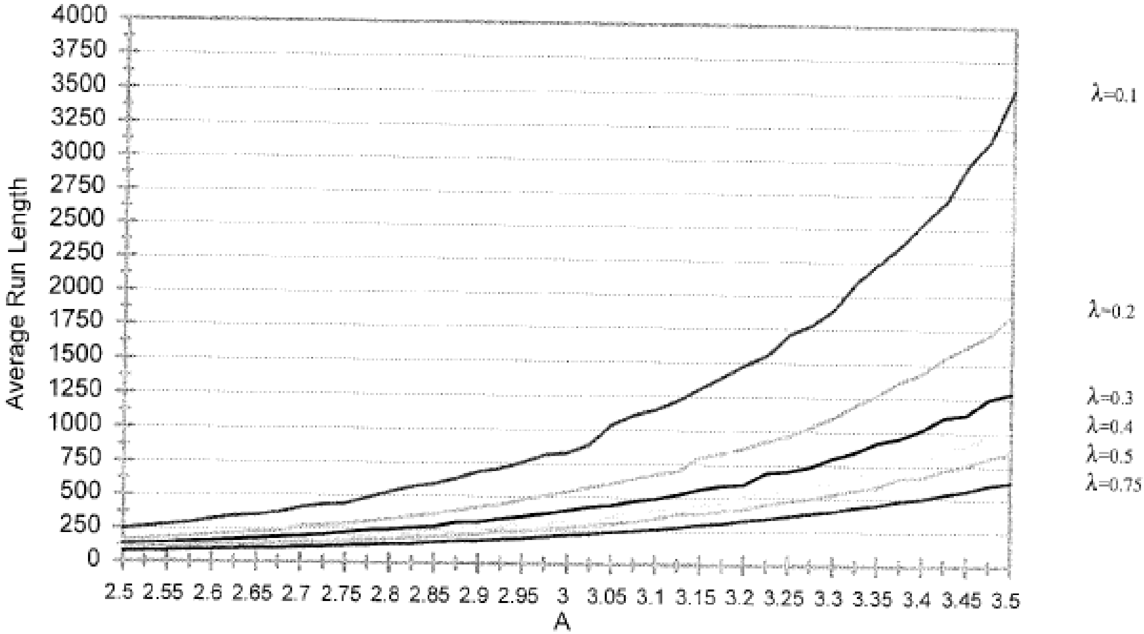


FIGURE 6. ARL's For various Values of λ and A With an In-Control Mean of 8.0.

ARL for Counted Data EWMA (Borror, Champ, and Rigdon 1998)

μ_0=10.0 ARL_0 for a false positive signal.

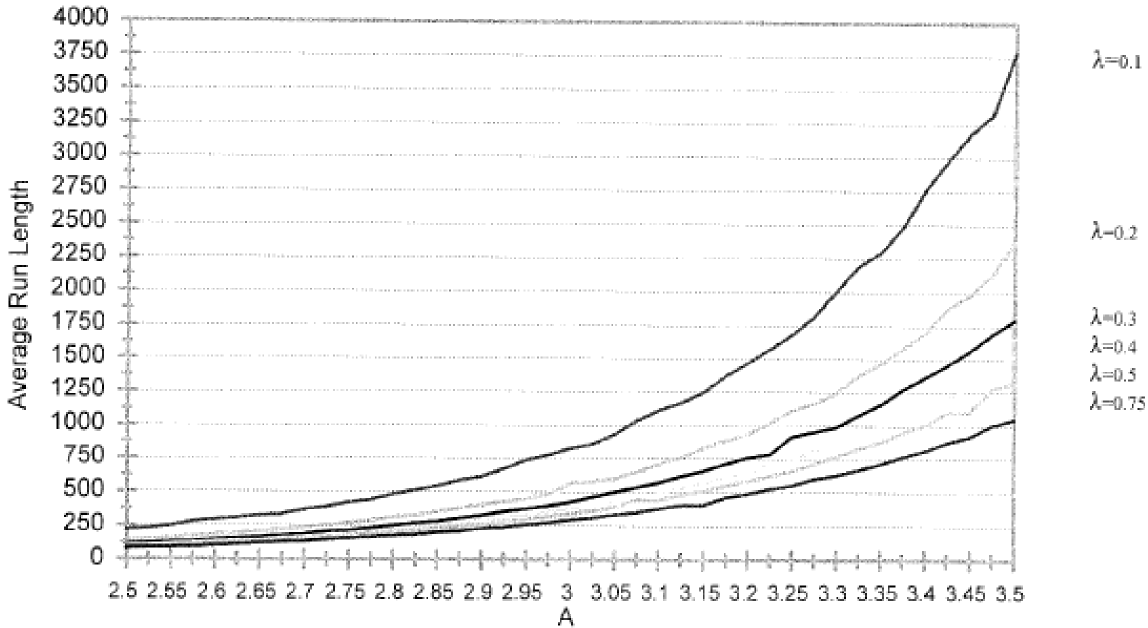


FIGURE 7. ARL's For various Values of λ and A With an In-Control Mean of 10.0.

ARL for Counted Data EWMA (Borror, Champ, and Rigdon 1998)

μ_0=15.0 ARL_0 for a false positive signal.

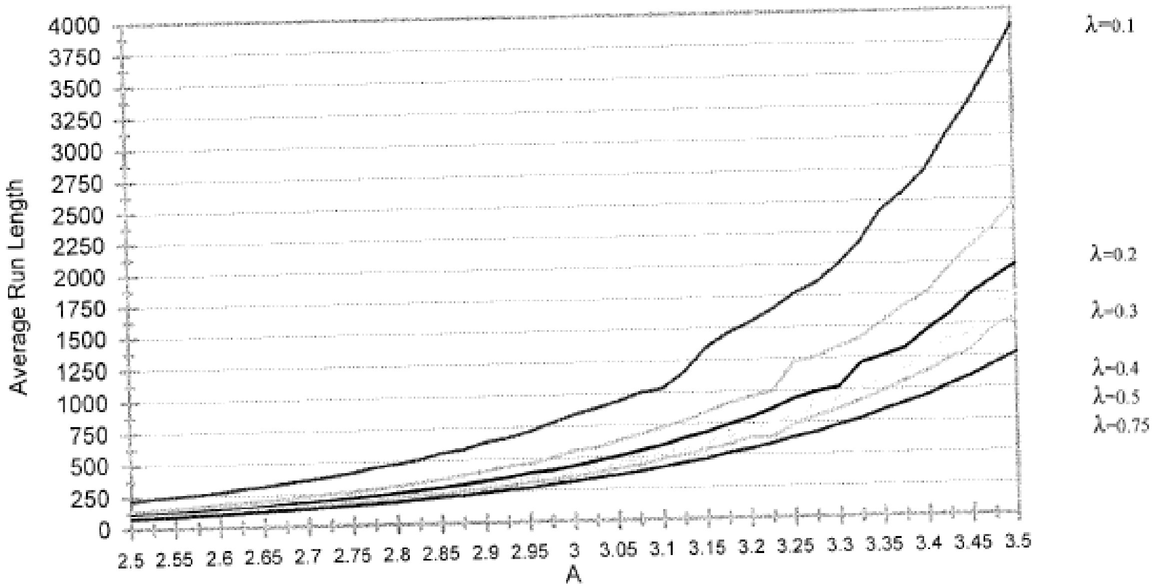


FIGURE 8. ARL's For various Values of λ and A With an In-Control Mean of 15.0.

ARL for Counted Data EWMA (Borror, Champ, and Rigdon 1998)

μ_0=20.0 ARL_0 for a false positive signal.

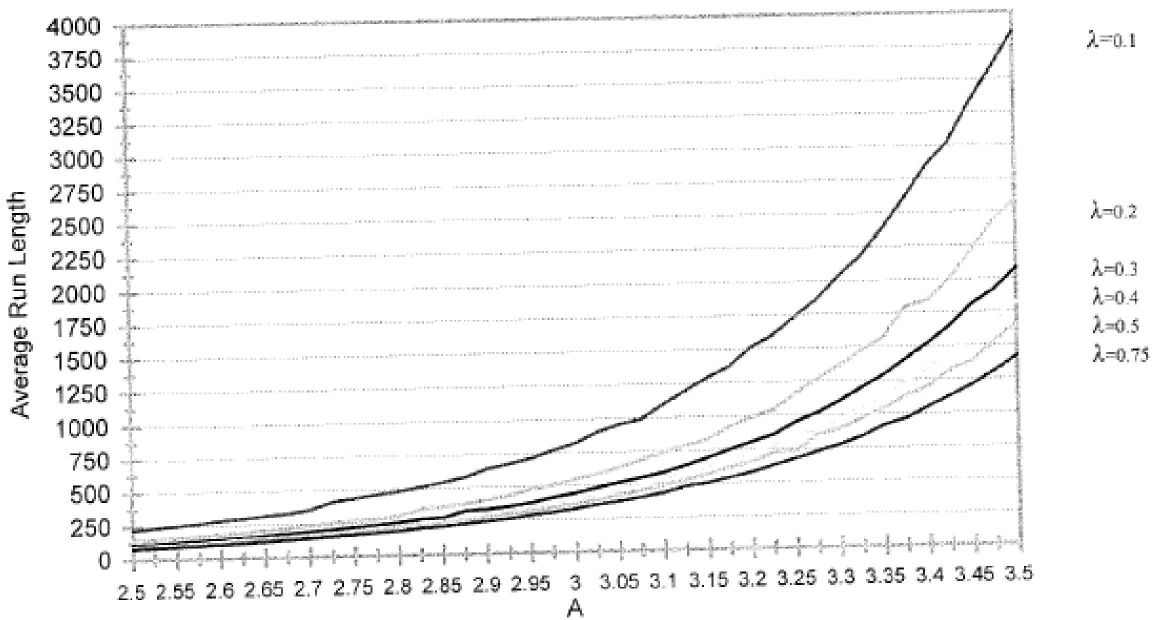


FIGURE 9. ARL's For various Values of λ and A With an In-Control Mean of 20.0.