Shewhart Control Charts in Phase I

August 15, 2020

Outline

Preferred means of Preventing Nonconformance

- MIL-STD-1916
  - ”sampling inspection alone inefficient for demonstrating conformance to requirements of a contract
  - ”statistical process control is the preferred means of preventing nonconformance”
- Later Civilian Standards
  - ASQ/ANSI/ISO 7870-2: 2011 Guide for Use of Shewhart Control Charts
  - ISO 22514-1:2014 Guide to Capability and Performance Studies
  - ASQ/ANSI/ISO 7870-4:2011 Guide for Cusum Control Charts
  - ASQ/ANSI/ISO 7870-6:2016 Describes EWMA Control Charts

Output of all Processes are subject to variability

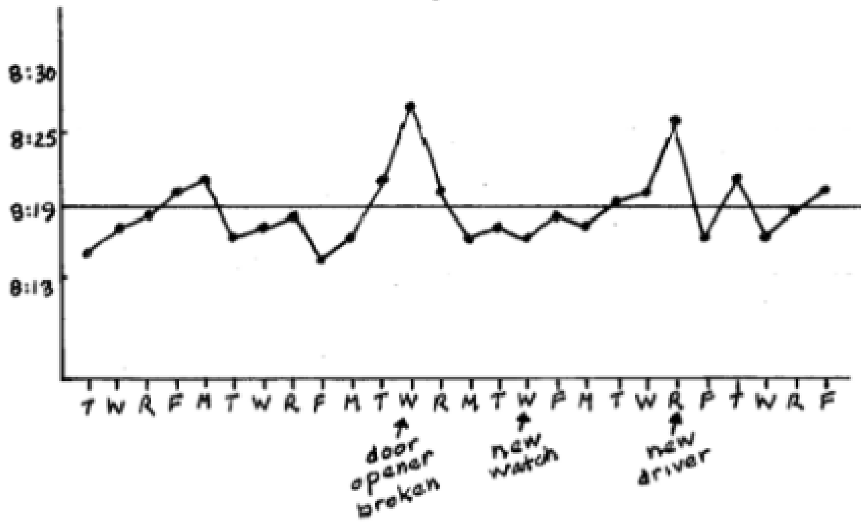
- Common Causes of Variability
  - Due to the inherent nature of the process
  - Can't be eliminated without changing the process
- Assignable (or Special) Causes of Variability
  - Can be Recognized
  - When removed by adjustments it leads to reduced variability in process output

Control Charts are Statistical Tools

- They are the most effective way of distinguishing between Common and Assignable Causes of Variability
- They can help Prevent overreaction to common causes (which may make things worse)
- They can help prevent ignoring Assignable Causes and potential reduction in process variability

A Simple Example

- 11 year-old Patrick Nolan Needed a Science Project
- His Father a Statistician suggested he collect data on something he care about
- He collected data on his school bus
- Arrival times each morning and notes about anything he considered unusual that day
- After a few weeks he constructed the following chart



Control Chart of Patrick's Data

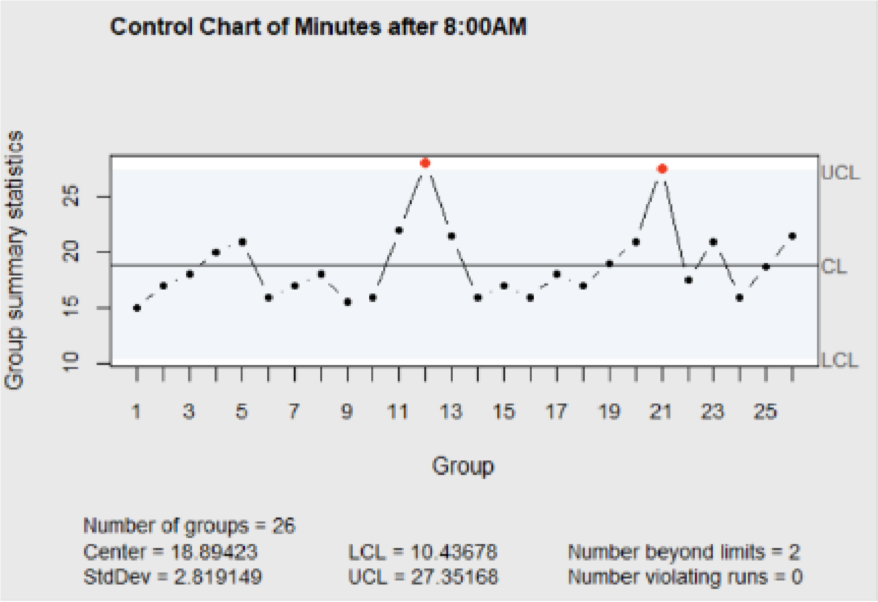


Figure 4.2 Control Chart of Patrick's Data

- Control Limits at $\pm$ 3 σ from mean
- Delayed times when door opener was broken and a new driver present were due to assignable causes
- The other unusual day noted by Patrick was within the limits

Situations where Control Charts are Used

There are two different situations where control charts are used.

1. Phase I- where initial control limits for the control chart are established from historical data

2. Phase II- where a control chart with predefined limits is used to monitor a process in real time to detect assignable causes

Types of Control Charts

The two types of control charts presented in this book are:

1. Shewhart Control Charts - used most effectively in Phase I
2. Time weighted Control charts - used most effectively in Phase II
3. This Chapter will discuss Shewhart Control Charts

Phase I

The purpose for developing control charts in Phase I is to:

- Establish the in-control process mean and variance and or control limits to be used in Phase II monitoring
- Develop and record an Out-of-control Action Plan (OCAP) to be used in Phase II

Phase I continued

Developing Control Charts in Phase I is usually an iterative process, and is best done using a computer.

1. Calculate control limits from a retrospective set of data
2. Plot the retrospective data on a control chart with limits determined in the first step
3. Identify out of control signals on the control chart
4. Search for the cause of out of control points (this can be a lengthy process!). Each cause identified should be added to the OACP for future reference in Phase II.
5. Eliminate out of control data points for which a cause is found.
6. Return to step one and continue iterating until a control chart showing no out of control points is found.

Phase II

In Phase II, the information obtained in Phase I (OCAP, process μ, σ are used to determine control chart limits for monitoring real time data. When an out of control signal is encountered, the OCAP is used to determine what adjustment is needed to bring the process back into control.

Phase II

In order to use control charts effectively to remove assignable cause variability in Phase II monitoring, the Automotive Industry Action Group recommends the following preparatory steps be taken.

- Establish an environment suitable for action
- Define the process
- Determine the characteristics to be charted
- Define the measurement system

Data for Shewhart Control Charts

Data for Shewhart Control Charts is collected in rational subgroups

- Variability of data within a rational subgroup should only be due to common causes of variability
- Variability in subgroup means should only be due to assignable causes of variability
- The two points above a can usually be accomplished by grouping consecutive items produced by the process together in a rational subgroup, then waiting long enough for a potential assignable cause to occur before collecting another subgroup.
- Subgroup size for variable control charts is typically 4-5 items in a subgroup, larger subgroup sizes can be used for $\overline{X} - s$ control charts.
- 25 or more subgroups are required for Phase I.

Shewhart $\overline{X} - R$ Control Charts

- $\overline{X}$-charts are used for monitoring the process mean.
- R-charts are used for monitoring the process variability
- $\overline{\overline{X}}$ is the average of all subgroup means, and $\overline{R}$ is the average of all subgroup ranges.
- Control limits for the $\overline{X}$-chart are $\overline{\overline{X}} \pm A_2\overline{R}$.
- Control limits for the R-chart are $D_4\overline{R}$
- The constants A_2 and D_4 are functions of the subgroup size and are given in the vignette `SixSigma::ShewhartConstants` in the R package `SixSigma`. They can also be found in Section 6.2.3.1 of the online NIST Engineering Statistics Handbook.

Example 1

Consider the retrospective data on Coil Resistance values from Mitra(1998)- 25 subgroups of 5

Table 4.1 Coil Resistance

Subgroup	Ohms				
1	20	22	21	23	22
2	19	18	22	20	20
3	25	18	20	17	22
4	20	21	22	21	21
5	19	24	23	22	20
6	22	20	18	18	19
7	18	20	19	18	20
8	20	18	23	20	21
9	21	20	24	23	22
10	21	19	20	20	20
11	20	20	23	22	20
12	22	21	20	22	23
13	19	22	19	18	19
14	20	21	22	21	22
15	20	24	24	23	23
16	21	20	24	20	21
17	20	18	18	20	20
18	20	24	22	23	23
19	20	19	23	20	19
20	22	21	21	24	22
21	23	22	22	20	22
22	21	18	18	17	19
23	21	24	24	23	23
24	20	22	21	21	20
25	19	20	21	21	22

Example 1 continued

```
Coil - Notepad
File  Edit  Format  View  Help
Obs1, Obs2, Obs3, Obs4, Obs5
20,   22,   21,   23,   22
19,   18,   22,   20,   20
25,   18,   20,   17,   22
```

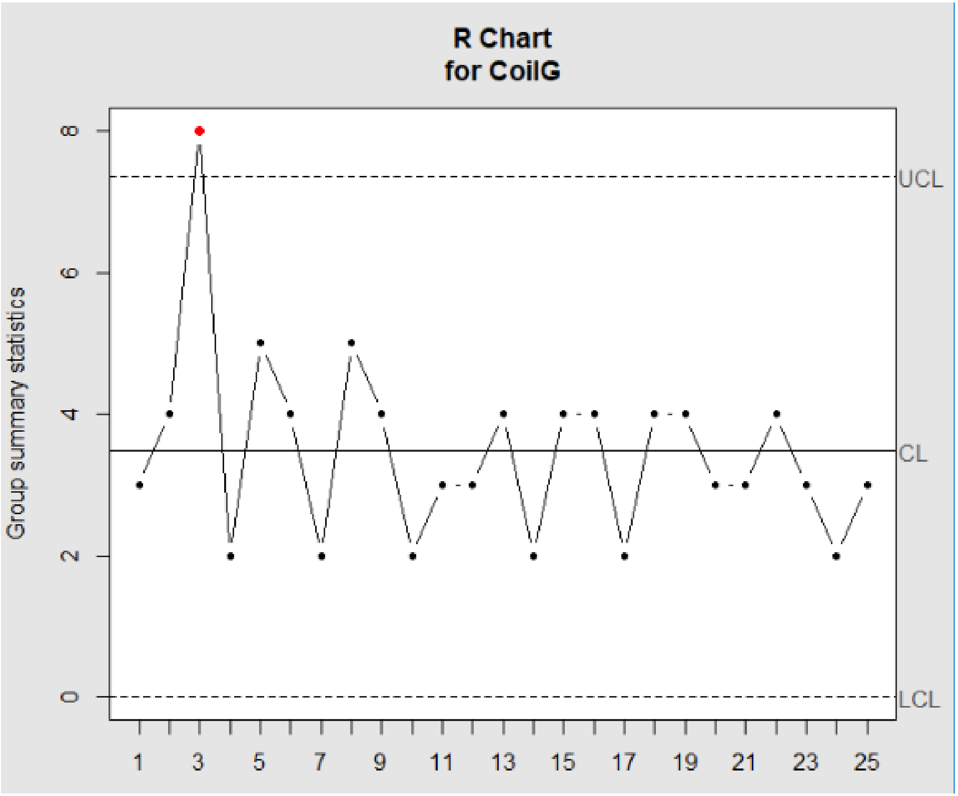
Figure 4.1 Coil.csv file

The data is read into an R data frame using the commands shown below, and the **qcc** function in the R package **qcc** is used to create the R-chart shown in Figure 4.2.

```r
#Example from Mitra
CoilG <- read.table("Coil.csv", header=TRUE,sep=",",na.strings="NA",dec=".",strip.white=TRUE)
library(qcc)
qcc(CoilG, type="R")
```

Example 1 continued

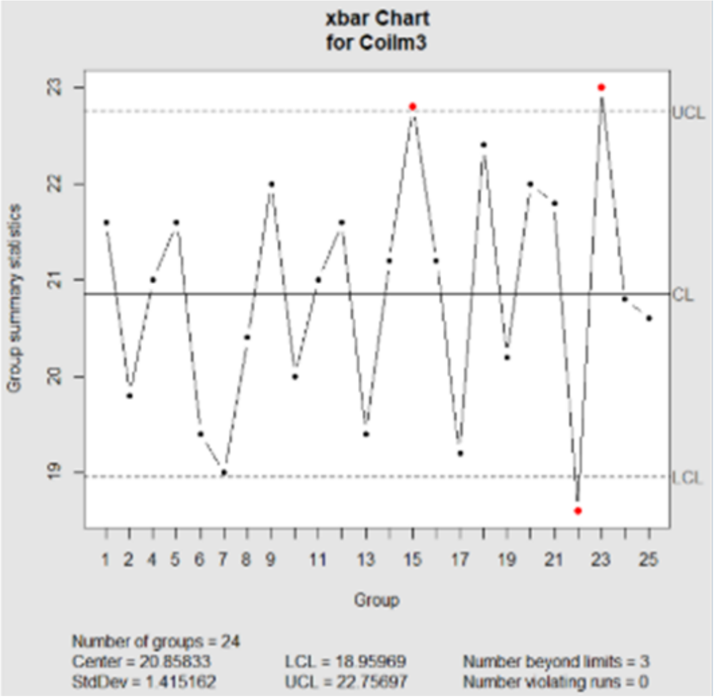
Begin Phase I looking at the R chart. If there are out of control points on the R chart, $\overline{R}$ will be too large and the control limits on the $\overline{X}$ chart will be too wide.



Example 1 continued

The high variability in the 3rd subgroup was caused by a new vendor of raw materials. A new policy requires new vendors to provide documentation showing their process could meet the requirements, therefore subgroup 3 was removed. The reconstructed R chart had no further out of control signals. Next the $\overline{X}$ bar chart was constructed.

```
# Eliminate subgroup 3
library(qcc)
Coilm3<-CoilG[-c(3), ]
qcc(Coilm3, type="xbar")
```



Example 1 continued

The oven temperature was too high when producing subgroup 22, and the wrong die was used when producing subgroup 23. No cause could be found for the high average in group 15. Eliminating groups 22 and 23 the limits were recomputed for the $\overline{X}$ chart. Subgroup 15 was still out of the limits.

The next step would be to eliminate subgroup 15 and add additional subgroups to bring the total number back to 25 or more, then recompute the limits again.

It may take many iterations until a control chart showing no out of control signals results.

Example 1 continued

Based on the causes for out of control signals found in this Phase I example, The OCAP should be updated as follows.

OCAP

Out of Control on R chart

1. Verify that the vendor of raw material has documented their ability to supply within specs. If not switch vendors.

Out of Control on $\overline{X}$ chart

1. Check to make sure proper die was used and if not switch to the proper die
2. Check to make sure temperature was set correctly, if not reset the temperature

Effective Rational Subgroups

- Rational subgroups are effective when common causes for variation occur withing subgroups
- Rational subgroups are effective when assignable causes for variation occur between subgroups
- When assignable causes for variation in the mean are detected on a control chart it indicates the rational subgroups were chosen effectively
- When all points on an $\overline{X}$ chart hug the centerline (within $\pm$ 1 σ of the mean) it indicates poor grouping and assignable cause variation occurs within subgroups
- with assignable cause variation within subgroups it will be difficult to detect assignable causes with the control chart

Discovery of the reasons for Assignable Causes

- Not always as obvious as Patrick's Chart or this Example
- Ideas for finding the reasons for assignable causes is presented later in this chapter

Interpreting Charts for Assignable Cause Signals

In addition to just checking individual points against the control limits to detect assignable causes, a list of additional indicators that should be checked.

1. A point above or below the upper control limit or below the lower control limit
2. Seven consecutive points above (or below) the center line
3. Seven consecutive points trending up (or down)
4. Middle one-third of the chart includes more than 90% or fewer than 40% of the points after at least 25 points are plotted on the chart
5. Obviously non-random patterns

These items are based on the Western Electric Rules proposed in the 1930s. If potential assignable cause signals are investigated whenever any one of the indicators are violated (with the possible exception of number 3.), it will make the control charts more sensitive.

$\overline{X} - s$ Charts

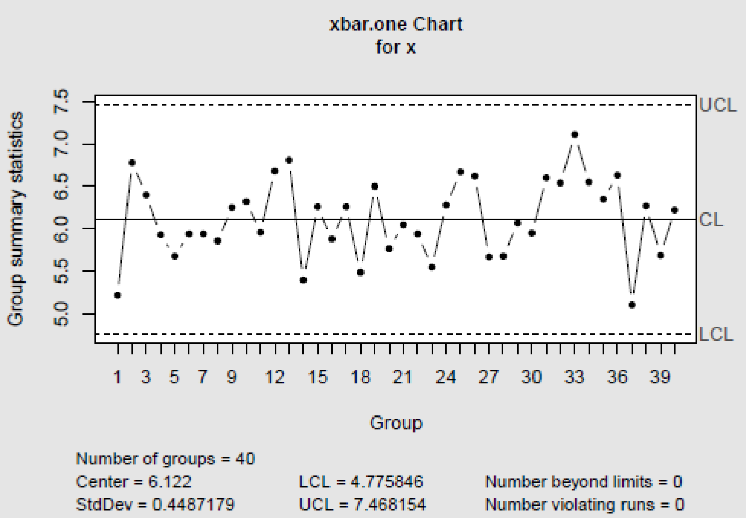
The $\overline{X}$ and s Charts can also be computed by the `qcc` function by changing the option `type="R"` to `type="S"` in the function call for the R -chart, and in adding the option `std.dev=MVALUE-STD` to the call for the $\overline{X}$-chart.

When the subgroup size is 4 or 5, as normally recommended in Phase I studies, there is little difference between $\overline{X}$ and s charts and $\overline{X}$ and R Charts, and it doesn't matter which is used. For larger subgroup sizes $\overline{X}$ and s charts should be used.

X and moving Range Chart

The pH of the output of a continuous chemical process is measured every 15 minutes (no rational subgroups). The data from one 10 hour period is shown in the code and output below. The standard deviation is calculated as $\hat{\sigma} = \overline{R}/d_2$, where $\overline{R}$ is the average moving range of 2 and d_2 is obtained from the table of factors for control charts with $n = 2$.

```
library(qcc)
x<-c(5.22,6.78,6.40,5.93,5.68,5.94,5.94,5.86,6.25,6.32,5.96,6.68,6.81,5.40,6.26,5.88,6.26,
     5.49,6.50,5.77,6.05,5.94,5.55,6.28,6.67,6.62,5.67,5.68,6.07,5.95,6.60,6.54,7.11,6.55,
     6.35,6.63,5.11,6.27,5.69,6.22)
qcc(x,type="xbar.one",method=MR)
```



Shewhart Attribute Charts used in Phase I

While variable control charts track measured quantities related to the quality of process outputs, attribute charts track counts of nonconforming items or the number of nonconformities. When individual items are inspected and each item can only be classified as conforming or nonconforming, attribute charts for the number or proportion of nonconforming in a subgroup are used. When each item inspected may contain 0, 1 or more nonconformities, attribute charts for the number of nonconformities are used.

- Attribute charts are not as informative as variables charts for Phase I studies.
- A shift or run of points on a variables chart may give a hint about the cause.
- A shift or run of the number nonconforming on an attribute chart may give no such hint.

However, in service industries and other non-manufacturing areas, counted data may be abundant while numerical measurements are rare.

Types of Shewhart Attribute Control Charts

The types of attribute charts we will discuss are:

- p-charts for the the proportion nonconforming
- np-charts for the number nonconforming in groups of n
- c-charts for the number of nonconformities
- U-charts for the average number of nonconformites per standardize inspection unit.

p-charts

The control limits for a Shewhart p-chart are based on the normal approximation to the Binomial distribution and are given by the formulas below.

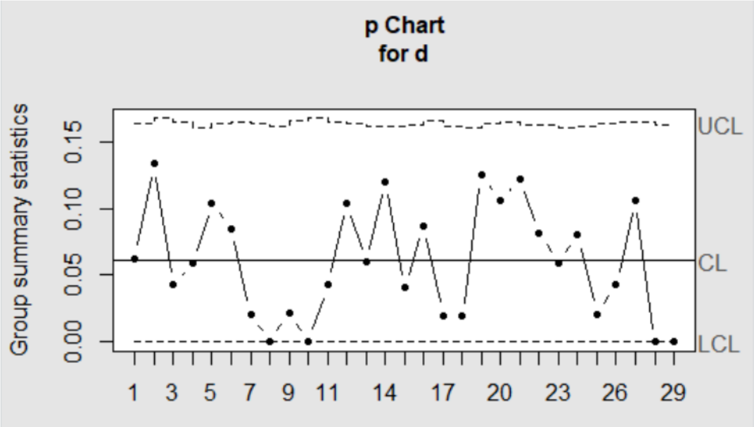
$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$
$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}},$$

where $\overline{p}$ is the average proportion nonconforming per subgroup in the Phase I data, and n is the number of items in a subgroup. n is usually large so that that np will be greater than 5 to justify the normal approximation.

Creating a p-chart with qcc

The following code creates a p-chart. `d` in the code represents the number nonconforming, and `sizes=n` specifies the number of items in a subgroup (n). Notice the control limits are not constant as calculated on page 199 of Christensen, Betz, and Stein(2013) since n varies from subgroup to subgroup.

```
library(qcc)
#Christensen's Table 14.1
d<-c(3,6,2,3,5,4,1,0,1,0,2,5,3,6,2,4,1,1,6,5,6,4,3,4,1,2,5,0,0)
n<-c(48,45,47,51,48,47,48,50,46,45,47,48,50,50,49,46,50,52,48,
     47,49,49,51,50,48,47,47,49,49)
qcc(d,sizes=n,type="p")
```



Example 2 - use of a p-chart in Phase I

The `qcc` package contains data from a Phase I study concerning the number of nonconforming cardboard frozen orange juice cans. The data can be retrieved as shown in the code below.

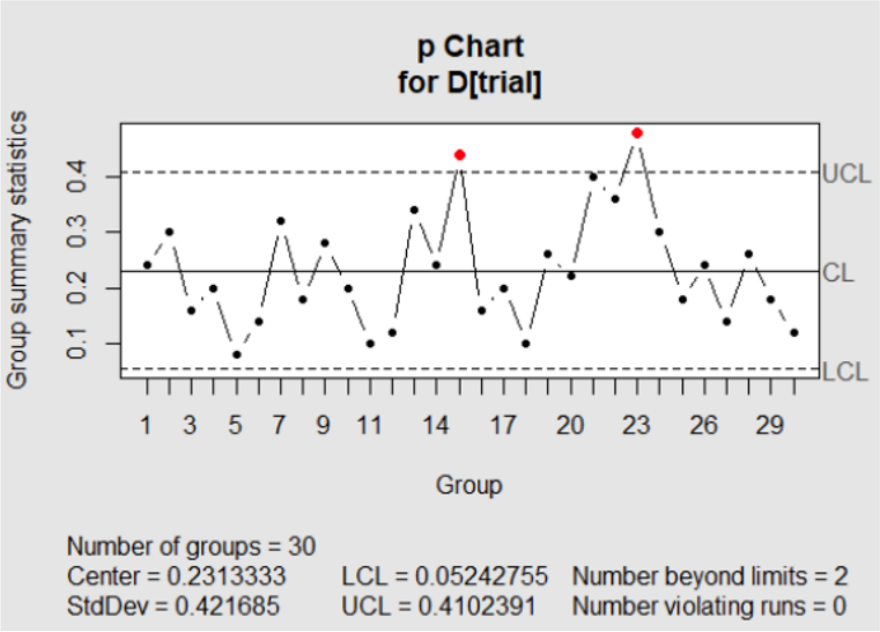
```
library(qcc)

data(orangejuice)

attach(orangejuice)

orangejuice

   sample  D size trial

1       1 12   50  TRUE

2       2 15   50  TRUE

3       3  8   50  TRUE

        .

        .

        .
```

The variable `trial` in the data frame takes on the values `TRUE` for the initial 30 values and `FALSE` for later follow-up observations.

Example 2 continued

The code below creates a p-chart, the `[trial]` in the code restricts the data used in creating the control chart to the first 30 values where `trial=TRUE`.

```
library(qcc)
qcc(D[trial], sizes=size[trial], type="p")
```



Example 2 continued

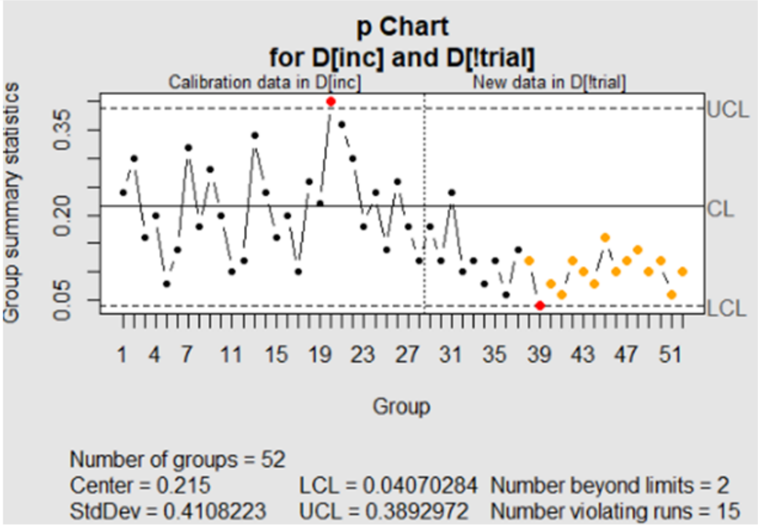
The causes for the high proportion nonconforming cans in subgroups 15 and 23 were found to be a new batch of cardboard, and an inexperienced operator. Removing those points and reconstructing the control chart revealed subgroup 21 above the upper control limit, but no specific cause for this could be found.

However, the average proportion nonconforming cans is 0.215, which is not an acceptable quality level (AQL). The plant management agreed, and asked the engineering staff to analyze the process to see if any improvements could be made. The study indicated that several adjustments could be made to the machine that should improve its performance.

Example 2 continued

After making these adjustments, 24 more subgroups (numbers 31-54) of n=50 were collected. The R code below eliminates runs 15 and 23 then plots the fraction nonconforming for these additional subgroups on the chart with the revised limits.

```
library(qcc)
# remove out-of-control points (see help(orangejuice) for the reasons)
inc <- setdiff(which(trial), c(15,23))
qcc(D[inc], sizes=size[inc], type="p", newdata=D[!trial], newsizes=size[!trial])
detach(orangejuice)
```



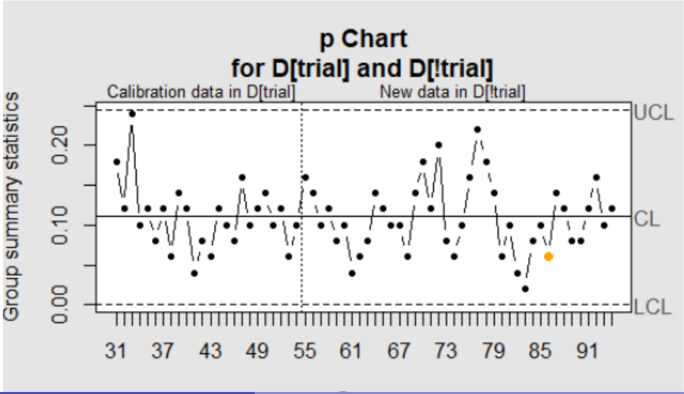
Example 2 continued

The new chart indicates an assignable cause at observation 38, due to the fact that there are 7 consecutive points below the center line starting at observation 32. This is a violation of the Western Electric rule 2.

Example 2 continued

The immediate impression from the new chart is that the process, after adjustments, is operating at a new lower proportion defective. Based on this improved performance, the control limits should be recalculated based on the new data. This was done, and additional data was collected over the next 5 shifts of operation (subgroups 55 to 94).

```
library(qcc)
data(orangejuice2)
attach(orangejuice2)
names(D) <- sample
qcc(D[trial], sizes=size[trial], type="p", newdata=D[!trial], newsizes=size[!trial])
detach(orangejuice2)
```



Example 2 continued

In this chart there is no indication of assignable causes in the new data. The proportion nonconforming varies between 0.02 to 0.22 with an average proportion nonconforming of 0.1108. This is not an acceptable quality level. However, due to the fact that no assignable causes are identified in Figure 4.6, the p chart alone gives no insight on how to improve the process.

Additional steps to identify assignable causes

1. Classify nonconforming cans & display in Pareto Chart
2. Scatter plots of $\hat{p}$ vs recorded processing conditions
3. After the above conduct group Brainstorming Sessions
4. Try PDCA & DoE to discover causes.

More detail on these steps are presented in the next section.

np-charts

np charts are preferred over p-charts when the subgroup size n is constant. Then the number nonconforming can be plotted on the chart rather than the proportion nonconforming. The formulas for the control limits are again based on the normal approximation to the Binomial distribution.

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$\texttt{Center line} = n\bar{p}$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

```
library(qcc)

d<-c(9,12,13,12,11,9,7,0,12,8,9,7,11,10)

qcc(d,sizes=1000,type="np")
```

c-charts

c-charts are appropriate for charting the number of nonconformities for constant size inspection units. The control limits are based on the normal approximation to the Poisson.

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$\texttt{Center line} = \bar{c}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

```
library(qcc)

data(circuit)

attach(circuit)

qcc(circuit$x[trial], sizes=circuit$size[trial], type="c")
```

c-charts

u-charts are appropriate for charting the average number number of nonconformities for a standardized inspection unit. The control limits are based on the normal approximation to the Poisson.

$$UCL = \overline{u} + 3\sqrt{\overline{u}/k}$$

$$\texttt{Center line} = \overline{u}$$

$$LCL = \overline{u} + 3\sqrt{\overline{u}/k}$$

```
library(qcc)
d<-c(6,7,8,8,6,7,7,6,3,1,2,3,3,4)
k<-c(12,10,8,9,8,9,8,10,10,10,9,12,10,12)
qcc(d,sizes=k,type="u")
```

7 tools[1] for discovering causes for out of control signals (Special Cause) with current data

- Flow Charts
- Cause and Effect Diagrams
- Check sheets - or Quality Information System
- Line Graphs or Run Charts
- Pareto Diagrams
- Scatter Plots
- Control Charts

These are used in conjunction with the PDCA (plan-do-check-act) cycle and Designed experiments (Chapter 5)

[1] Ishakawa

Flow Charts

Flow Charts are useful for helping everyone understand the proper process steps

- Out-of-control signals may results when the proper process steps are not followed.
- Process Flow Charts should be developed as a group exercise by those who work in the process
- The initial Flow Chart is developed by consensus
- Validity of the Flow Chart Should be tested in practice
- The Flow Chart should be modified when process result does not meet the desired level

Flow Charts

Flow Chart Symbols

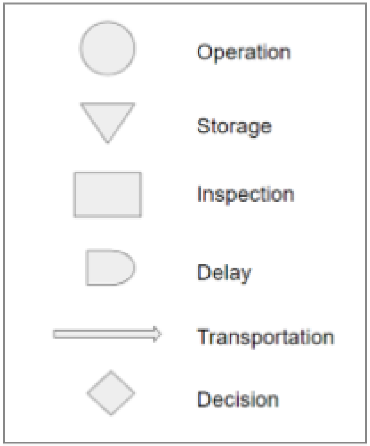
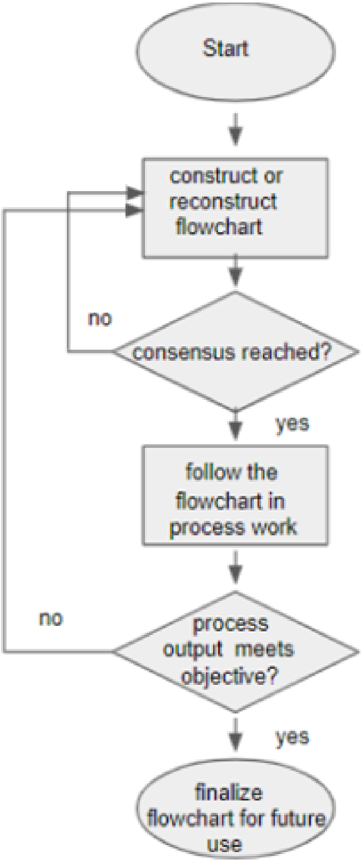


Figure 4.9 Flowchart Symbols

Example Flow Chart



PDCA or Shewhart Cycle

To determine the reason for, or corrective action for, an assignable cause you have to do something not just observe process results

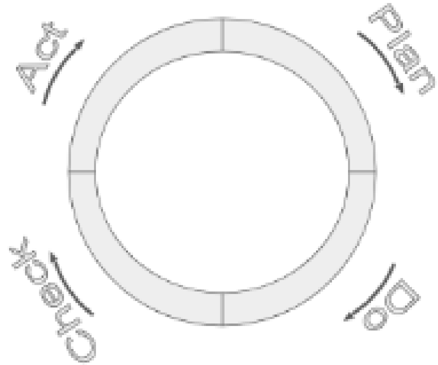


Figure 4.11 PDCA or Shewhart Cycle

Example: Investigate whether failure to follow a process step was the cause of an out-of-control signal on a control chart

PDCA or Shewhart Cycle

1. Plan - in this step a plan would be made to investigate the potential cause of the out-of-control signal
2. Do - in this step the the modified process step that was followed at the time of the out-of-control-signal will be purposely followed again at the present time.
3. Check - in this step the process output that is measured and recorded on a control chart will be checked to see if another out-of-control signal results after the purposeful change in procedure.
4. Act - if the out-of-control signal returns, it can be tentatively assumed to have been caused by the purposeful change made in step 2. The "Act" taken should then be to prevent the change that was made from occurring in the future.

PDCA or Shewhart Cycle

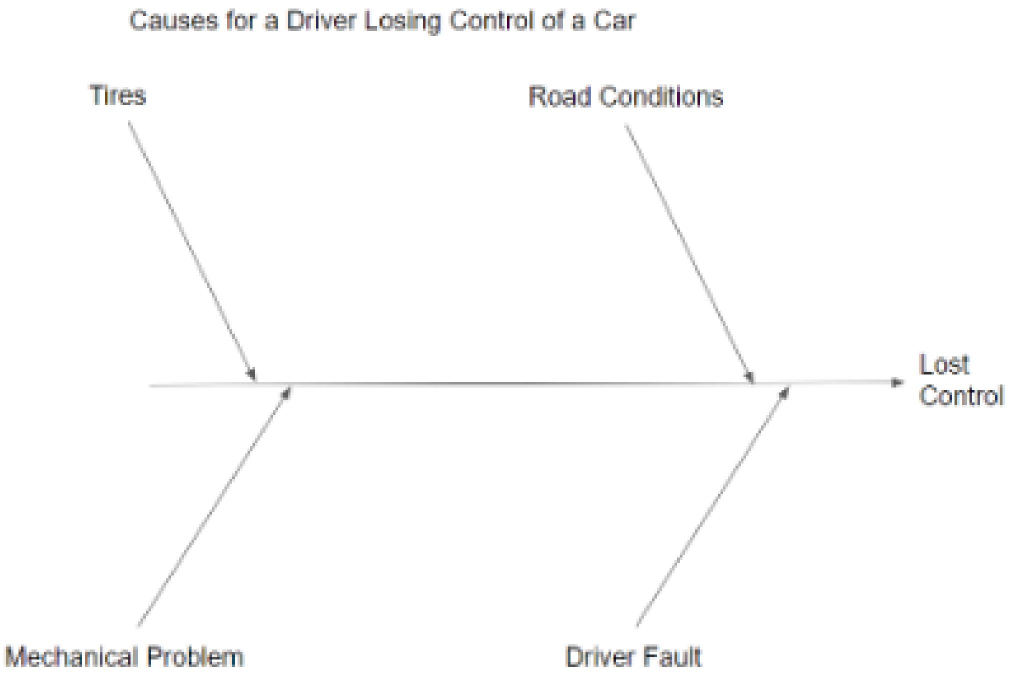
The PDCA Cycle is similar to the steps in the scientific method, and is often repeated several times to reach a solution.

TABLE 4.2: PDCA and the Scientific Method

Step	PDCA	Scientific Method
1	Plan	Construct a Hypothesis
2	Do	Conduct an Experiment
3	Check	Analyze data and draw Conclusions

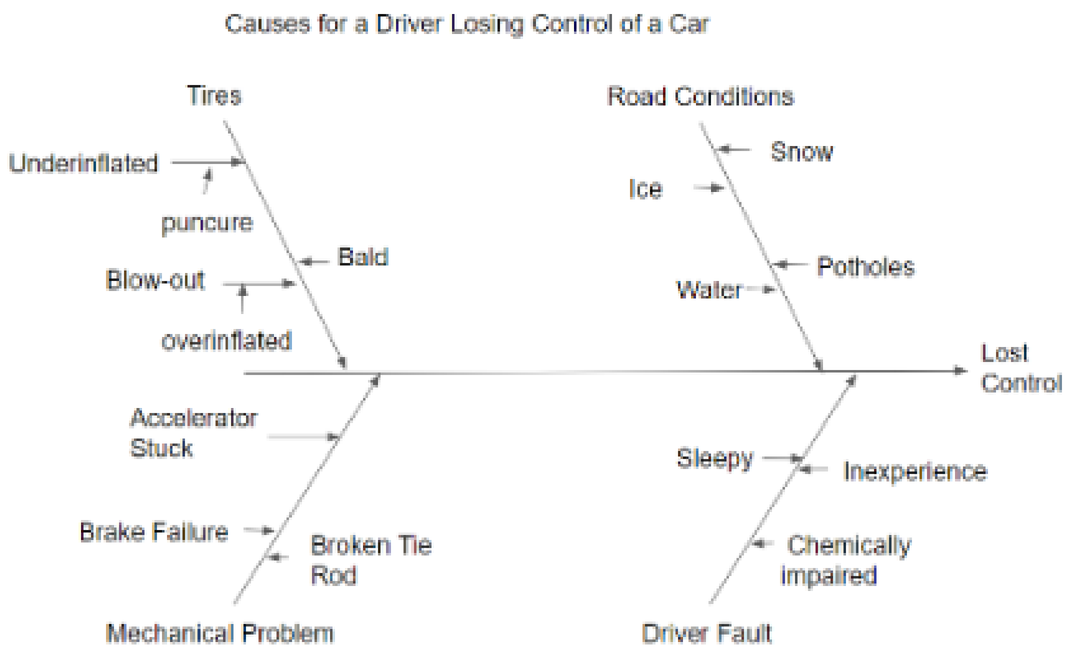
Cause and Effect Diagrams

- Used to collect opinions regarding the cause of a process problem
- Usually made in a group setting of those familiar with the process
- Opinions are organized on a diagram showing major categories



Cause and Effect Diagrams

Details can be added as shown



Cause and Effect Diagrams

Can be produced with R

```
library(qcc)
nonconformities<-c(2,30,5,1,9,3)
names(nonconformities)<-c("Contamination","Sealing Failure",
  "Adhesive Failure","Material Defects","Printing Off Color",
  "Ink Migration")
paretoChart(nonconformities,
    ylab = "Nonconformance frequency",main="Pareto Chart")
```

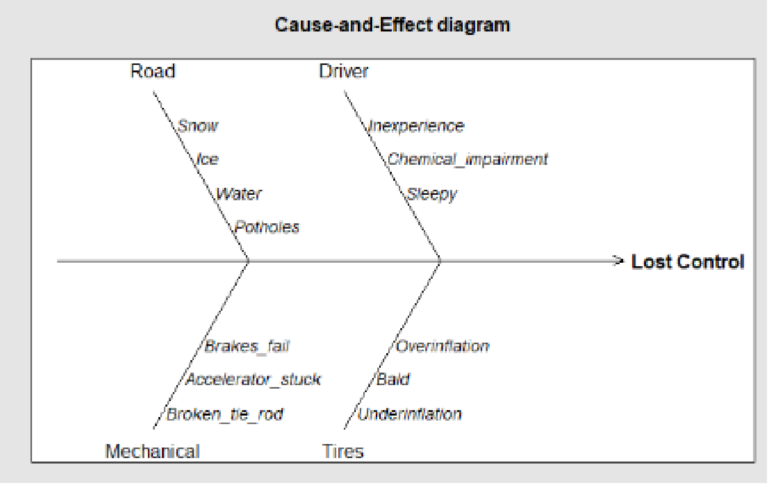


Figure 4.15 qcc Cause-and-Effect Diagram

Check Sheets or Quality Information System

Check Sheets were devised so that people involved in work within a process could quickly record information on paper to be filed for future reference. Below is a Defective item Check Sheet.

Check Sheet

Product:

Date:

Location:

Manufacturing Stage:

Number inspected:

Inspector's name

Remarks:

Lot No.

Order No.

Type	Check	Subtotal
Contamination	//	2
Sealing Failure	~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~	30
Adhesive Failure	~~////~~	5
Material Defects	/	1
Printing Off Color	~~////~~ ////	9
Ink Migration	///	3
	Grand Total:	50

Check Sheets or Quality Information System

Other types of Check Sheets defined by Ishikawa are

- Defect Location
- Defect Cause
- Check-up confirmation
- Work Sampling

However, in today's world, having records stored on paper in file cabinets is obsolete technology and has been replaced by computer files

Check Sheets or Quality Information System

A Quality Information System (QIS) is the modern equivalent of check sheets filed by date stored in a computer database.

A QIS contains quality-related records from customers, suppliers and internal processes. It can be used to

- Record critical data (e.g. measurement and tool calibration)
- Store quality process and procedures
- Create and deploy operating procedures (e.g. ISO 9001 - based quality management system)
- Document required training
- Initiate action (e.g. generate a shop order from a customer order)
- Control processes
- Monitor processes real time control charting
- Schedule resource usage
- Manage a knowledge base
- produce routine reports or “dashboards” for managers to give current information about what has happened and what is currently happening.

Check Sheets or Quality Information System

- The data in a QIS is often stored in relational databases and organized in a way so that portions of it can be easily be retrieved for special purposes
- QIS are often required e.g., the FDA requires pharmaceutical companies to maintain a QIS

Line Graphs or Run Charts

To determine the cause of a assignable cause pattern seen on a control chart, it is often useful to look for similar patterns in variables in the QIS that represent process settings or conditions.

For example if a control chart showed improvement with a run of 7 points below the centerline, you might search for other process variables that have a similar pattern. For example the plot below shows a similar pattern in the recorded standard deviation of reference standards.

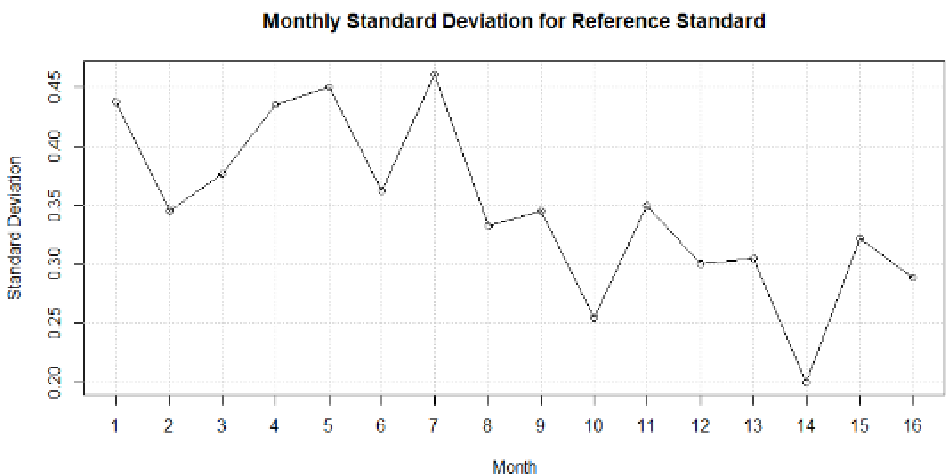


Figure 4.17 Standard Deviations of Plaque Potency Assays of a Reference Standard

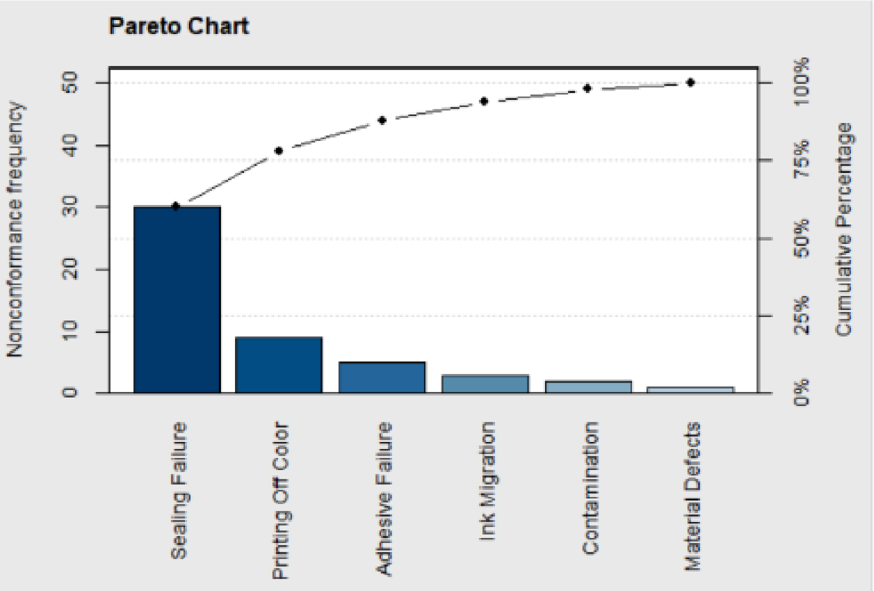
Line Graphs or Run Charts

Line graphs can be produced with the `plot()` command in R as shown below.

```
# page 105 (a) - produces Figure 4.17 on page 104
s<-c(.45,.345,.375,.435,.45,.36,.46,.335,.348,.252,.35,.30,.302,.20,.325,.29)
plot(s,type="o",lab=c(17,5,4),ylab="Standard Deviation",xlab="Month",
main = "Monthly Standard Deviation for Reference Standard")
grid()
```

Pareto Diagrams

When an Attribute Chart indicates an assignable increase in nonconformances or nonconformities, it is often difficult to determine the reason. Classifying the nonconformances or nonconfomities often helps. The Pareto diagram is a graphical way of representing a classification. For example, the Pareto Diagram below displays the subcategories of defective orange juice cans for subgroup 23 in Figure 4.6.



Pareto Diagrams

The Pareto Diagram is a bar graph with bar heights equal to the number of nonconformities in each category or classification. The bars are ranked from largest to smallest, and the scale on the left of the graph is for the count in each category and the scale on the right is for the cumulative percent. From the chart on the last slide it can be seen that over 50% of the defective cans were classified as sealing failures. Knowing that sealing failures are usually due to operator error, helped reduce these defects.

Pareto Diagrams can be produced by the `paretoChart()` function in the `qcc` package

```
library(qcc)
nonconformities<-c(2,30,5,1,9,3)
names(nonconformities)<-c("Contamination","Sealing Failure",
  "Adhesive Failure","Material Defects","Printing Off Color",
  "Ink Migration")
paretoChart(nonconformities, ylab = "Nonconformance
            frequency",main="Pareto Chart")
```

Scatter Plots

Rather than looking at many Line Graphs to find one that matches the pattern on the control chart of a quality characteristic, scatter plots can be used.

- Scatter plots are graphical tools that can be used to illustrate the relationship or correlation between two variables.
- A scatter plot is constructed by plotting an ordered pair of variable values on a graph.
- The variable on the x-axis usually represents a process variable that can be manipulated
- The variable on the y-axis is usually a measured variable that is to be predicted

Scatter Plots

Scatter plots are made with the `plot()` function in R

```
plot(file$cycle_time, file$yield,xlab=c("Cycle
    Time"),ylab=c("Yield"),type='p',pch=20)
```

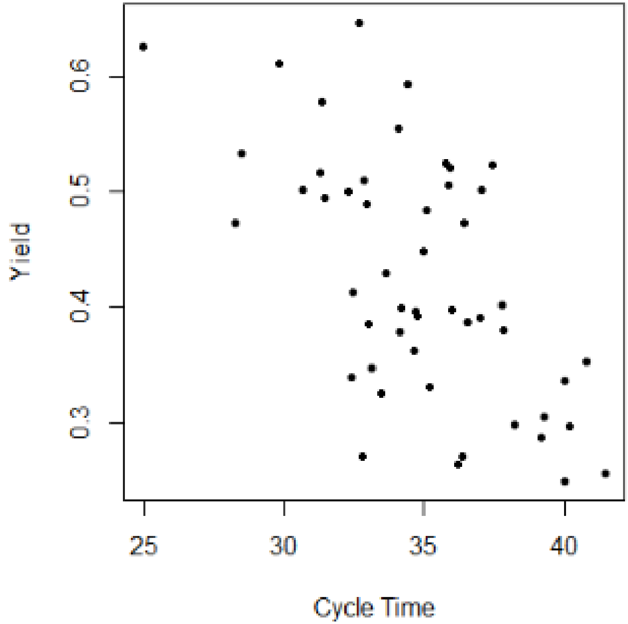


Figure 4.19 Scatter Plot of Cycle Time versus Yield

Process Capability Analysis

At the completion of a Phase I study, where control charts have shown that the process is in control at what appears to be an acceptable level, then calculating a process capability ratio (PCR) is an appropriate way to document the current state of the process.

When variables control charts were used in Phase I, then the PCR's C_p and C_{pk} are appropriate. These indices can be calculated with the formulas:

$$C_p = \frac{USL - LSL}{6\sigma}$$

$$C_{pk} = \frac{Min(C_{Pu}, C_{Pl})}{3\sigma}$$

Process Capability Analysis

$$C_{Pu} = \frac{USL - \overline{X}}{\sigma}$$

$$C_{Pl} = \frac{\overline{X} - LSL}{\sigma}$$

The measure of the standard deviation σ used in these formulas is estimated from the subgroup data in Phase I control charts.

Process Capability Analysis

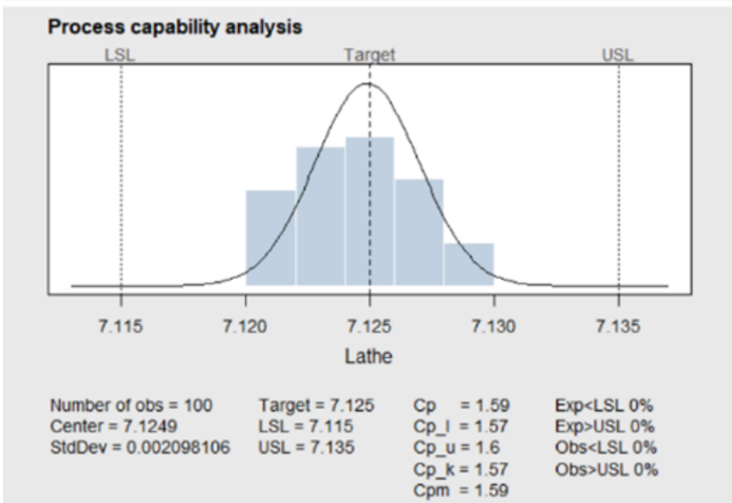
The `process.capability` function in the R package `qcc` can compute the capability indices and display a histogram of the data with the specification limits superimposed. For example, the R code below creates $\overline{X} - R$ charts using the data from Table 14.2 in Christensen et. al.(2013).

```
Lathe<-read.table("Lathe.csv",header=TRUE,sep=",",na.strings="NA",dec=".",strip.white=TRUE)
library(qcc)
qcc(Lathe,type="R")
pc<-qcc(Lathe,type="xbar")
process.capability(pc,spec.limits=c(7.115,7.135))
```

```
Process Capability Analysis

Number of obs = 100          Target = 7.125
       Center = 7.125           LSL = 7.115
       StdDev = 0.002098        USL = 7.135
```

Process Capability Analysis

```
Capability indices:

      Value   2.5%  97.5%
Cp    1.589  1.368  1.809
Cp_l  1.573  1.381  1.765
Cp_u  1.605  1.409  1.800
Cp_k  1.573  1.344  1.801
Cpm   1.587  1.367  1.807

Exp<LSL 0%   Obs<LSL 0%
Exp>USL 0%   Obs>USL 0%
```



Validity of C_p Index

The validity of the C_p index is dependent on the following:

1. The quality charateristic measured follows a normal distribution
2. The process is in a state of control
3. For two sided specification limits the process is centered

The control chart and output of the `process.capability` function serve to check these.

Capability Indices and Process Fallout in ppm

Table 4.2: Capability Indices and Process fallout in ppm

	Process Fallout (in ppm)		Process Fallout with 1.5σ shift	
PCR	One-sided Specs	Two-sided Specs	One-sided Specs	Two-sided Specs
0.50	66,807	133,614	500,000	501,350
1.00	1,350	2700	66,807	66,811
1.50	4	7	1,350	1,350
2.00	0.0009	0.0018	4	7

Table 4.2 makes it clear why Ford motor company began in 1980 requiring that their suppliers demonstrate their processes were in a state of statistical control and had a capability index of 1.5 or greater. That way Ford could be guaranteed an acceptable quality level (AQL) for incoming components from their suppliers, without the need for acceptance sampling.

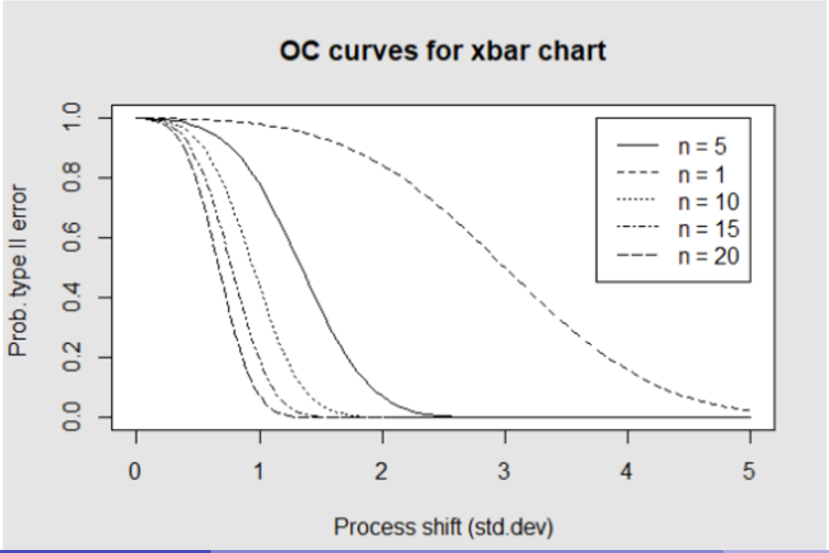
OC for Variable Control Charts

The operating Characteristic or OC for an $\overline{X}$-chart is defined as:

$$OC = \beta = P(LCL \leq \overline{X} \leq UCL)$$

It can be produced with the `oc.curves()` function as shown below.

```
Coilm <- read.table("Coil.csv", header=TRUE,sep=",",na.strings="NA",dec=".",strip.white=TRUE)
library(qcc)
pc<-qcc(Coilm, type="xbar",plot=FALSE)
beta <- oc.curves(pc, nsigmas=3)
```



ARL for Variable Control Chart

Given the mean shifts by $k\sigma$ just before the ith subgroup is plotted, the probability that the first mean to fall out of the control limits on an $\overline{X}$-chart is for the mth subgroup is

$$[\beta(\mu + k\sigma)]^{m-1}(1 - \beta(\mu + k\sigma)).$$

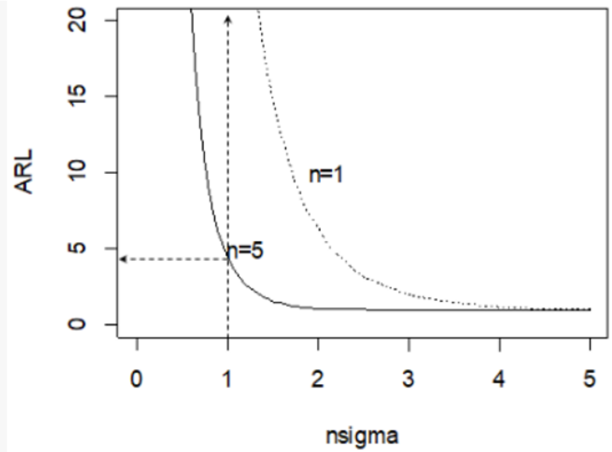
This is a Geometric probability with parameter $[\beta(\mu + k\sigma)]$, therefore the average time until a point falls our of the control limits called the ARL is given by

$$ARL = \frac{1}{(1 - \beta(\mu + k\sigma))}$$

The ARL Curve

The code below shows how to graph the ARL Curve for subgroup sizes 1 and 5.

```
library(qcc)
pc<-qcc(Coilm, type="xbar",plot=FALSE)
nsigma = seq(0, 5, length=101)
beta <- oc.curves(pc, nsigmas=3)
ARL5=1/(1-beta[ ,1])
ARL1=1/(1-beta[ ,2])
plot(nsigma, ARL1, type='l',,lty=3,ylim=c(0,20),ylab='ARL')
lines(nsigma, ARL5, type='l',lty=1)
text(1.2,5,'n=5')
text(2.1,10,'n=1')
```

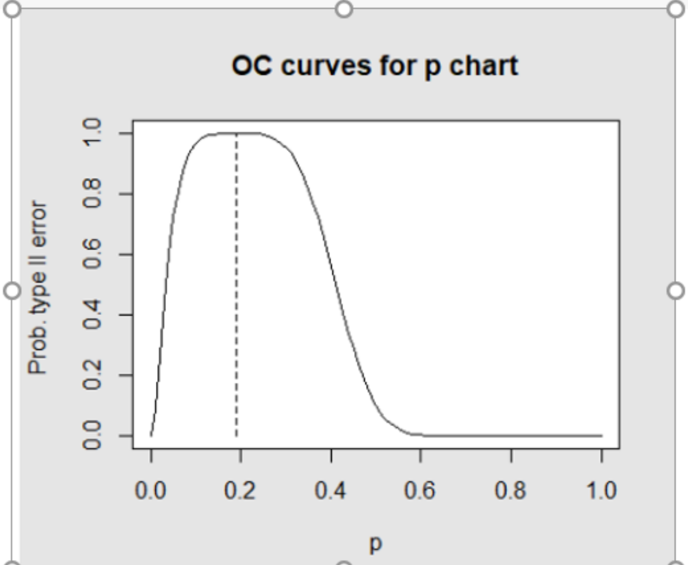


In the figure, it can be seen that if the subgroup size is $n = 5$, and the process mean shifts by 1 standard deviation, it will take 5 (rounded up to an integer) subgroups on the average before before the $\overline{X}$ chart shows an out of control signal.

OC and ARL Curves for Attribute Control Charts

The `oc.curves` function in the `qcc` package can also make and plot OC curves for p and c type attribute control charts. For these charts, β is computed using the Binomial or Poisson distribution. An example is shown below.

```
library(qcc)

data(orangejuice)

attach(orangejuice)

beta <- oc.curves(qcc(D[trial], sizes=size[trial], type="p"))
```



Summary

- The cause for out of control points is usually easier to find using variable control charts than attribute control charts
- That is important in Phase I studies where OCAP's are developed
- When the process is in control with $C_p \geq 1.5$ or $C_{pk} \geq 1.5$ no inspection of process output will be necessary

How to identify the cause of poor output and find corrective action

Depends on the type of problem

- **Special cause variation** - use typical tools (i.e. flow diagrams, C&E Diagrams, PDCA, and Pareto charts **with current data**) to identify the problem and test a solution
- **Common cause variation** and **Off target** - Study the entire process (here is where tools such as designed experiments Taguchi's parameter design can be very helpful)

The problem is that most don't recognize the difference between Special and Common Causes, and they react to everything as if it were a special cause. This is counterproductive!

Phase I and Phase II use of Control Charts

Control Charts are the **most effective** way to distinguish between common and assignable causes for variability.