Variable Sampling Plans

September 23, 2020

Outline

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- A sample size n

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- When using the k-method, an acceptance constant k

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- A sample size n
- When using the k-method, an acceptance constant k
- When using the M-method, the maximum proportion nonconforming M

k-method - lower specification limit, σ known

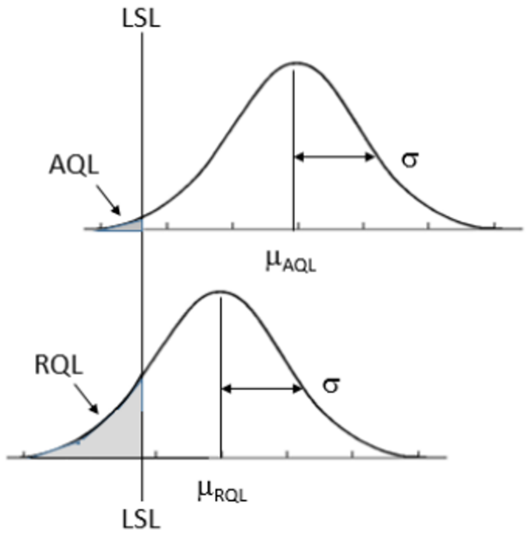


Figure 3.1: AQL and RQL for Variable Plan

Accepting the Lot is equivalent to failing to reject H_0: $\mu \geq \mu_{AQL}$ in favor of the alternative H_a: $\mu < \mu_{AQL}$

k-method - lower specification limit, σ known

If the producer's risk is α, and the consumers risk is β

$$P\left(\frac{\bar{x} - LSL}{\sigma} > k \mid \mu = \mu_{AQL}\right) = 1 - \alpha$$

$$P\left(\frac{\bar{x} - LSL}{\sigma} > k \mid \mu = \mu_{RQL}\right) = \beta.$$

or

$$P\left(Z > k\sqrt{n} + \frac{LSL - \mu_{AQL}}{\sigma/\sqrt{n}}\right) = 1 - \alpha,$$

and

$$P\left(Z < k\sqrt{n} + \frac{LSL - \mu_{AQL}}{\sigma/\sqrt{n}}\right) = \alpha.$$

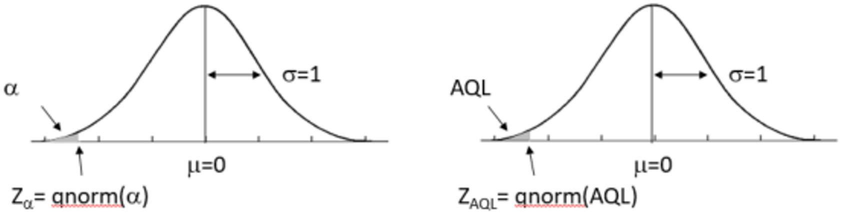
k-method - lower specification limit, σ known

$$k\sqrt{n}+\frac{LSL-\mu_{AQL}}{\sigma/\sqrt{n}}=Z_\alpha$$

or

$$k=\frac{Z_\alpha}{\sqrt{n}}-\frac{LSL-\mu_{AQL}}{\sigma}=\frac{Z_\alpha}{\sqrt{n}}-Z_{AQL},$$

$$\text{since}\ \frac{LSL-\mu_{AQL}}{\sigma}=Z_{AQL}.$$



k-method - lower specification limit, σ known

$$P\left(\frac{\bar{x} - LSL}{\sigma} > k \mid \mu = \mu_{RQL}\right) = \beta. \Rightarrow \qquad k = \frac{Z_{1-\beta}}{\sqrt{n}} - Z_{RQL}. \quad = \frac{Z_{\alpha}}{\sqrt{n}} - Z_{AQL},$$

$$\text{solving for } n \qquad n = \left(\frac{Z_{\alpha} - Z_{1-\beta}}{Z_{AQL} - Z_{RQL}}\right)^2$$

For the case where AQL=1% or 0.01, the RQL=4.6% or 0.046, $\alpha = 0.05$ and $\beta = 0.10$,

```
n<-((qnorm(.05)-qnorm(.90))/(qnorm(.01)-qnorm(.046)))^2
k<-(qnorm(.05) / sqrt(21))-qnorm(.01)
```

$\Rightarrow \qquad n = 20.8162. \qquad k = 1.967$

k-method - lower specification limit, σ known

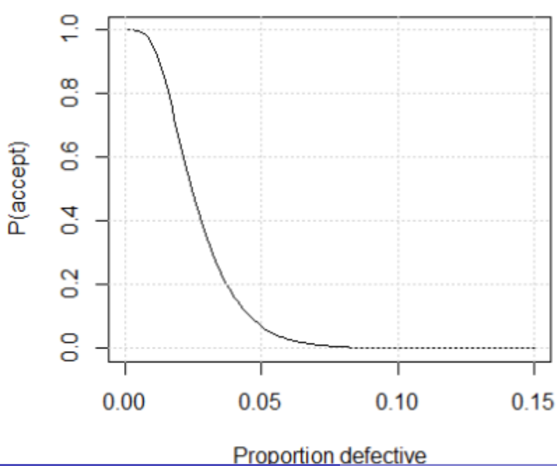
Thus, conducting the sampling plan on a lot of material consists of the following steps:

1. Take a random sample of n items from the lot
2. Measure the critical characteristic x on each sampled item
3. Calculate the mean measurement $\overline{x}$
4. Compare $(\overline{x} - LSL)/\sigma$ to the acceptance constant $k = 1.967411$
5. If $(\overline{x} - LSL)/\sigma > k$, accept the lot, otherwise reject the lot.

k-method - lower specification limit, σ known

The R-code below uses the `OCVar()` function in the `AcceptanceSampling` Package to store the plan and make the OC curve

```
library(AcceptanceSampling)
plnVkm<-OCvar(21,1.967411,pd=seq(0,.15,.001))
plot(plnVkm, type='l')
grid()
```



k-method - lower specification limit, σ known

The **find.plan()** function in the **AcceptanceSampling** automates the procedure of finding a sampling plan with a specific PRP and CRP

```
library(AcceptanceSampling)

find.plan(PRP=c(0.01, 0.95), CRP=c(0.046, 0.10), type="normal")

$n

[1] 21


$k

[1] 1.967411


$s.type

[1] "known"
```

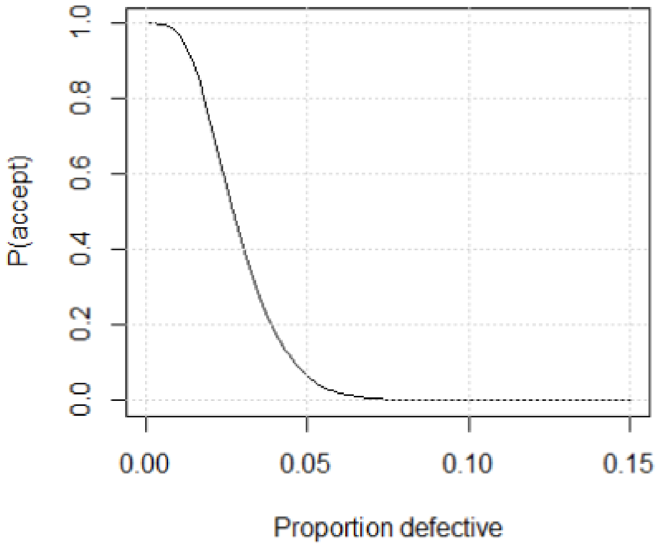
k-method - lower specification limit, σ known

The `find.plan()` function in the can find an equivalent Attributes sampling plan assuming that items can only be classified as above or below the lower spec limit.

```
library(AcceptanceSampling)
find.plan(PRP=c(0.01, 0.95), CRP=c(0.046, 0.10), type="binomial")
$n
[1] 172

$c
[1] 4

$r
[1] 5
```

k-method - lower specification limit, σ known

The OC curves for the Variable and Attribute plans are equivalent, but the sample size is much larger for the Attribute Plan (172 opposed to 21)



Example 1

The AQL is 0.02 or 2%, the RQL is 0.12 or 12%, the producers risk is $\alpha = 0.08$, the consumers risk $\beta = .10$, and the standard deviation $\sigma = 8$kg.

```
library(AcceptanceSampling)

find.plan(PRP=c(.02,.92), CRP=c(.12,.10), type='normal')

$n

[1] 10


$k

[1] 1.609426


$s.type

[1] "known"
```

Example 1 continued

After a sample of n=10 items $\overline{x} = 110$.

Example 1 continued

After a sample of n=10 items $\overline{x} = 110$.

$$Z_L = \frac{\overline{x} - LSL}{\sigma} = \frac{110 - 100}{8} = 1.25 < 1.609 = k$$

Example 1 continued

After a sample of n=10 items $\overline{x} = 110$.

$$Z_L = \frac{\overline{x} - LSL}{\sigma} = \frac{110 - 100}{8} = 1.25 < 1.609 = k$$

Therefore, reject the lot.

k-method - lower specification limit, σ unknown

solving for n $$n = \left(\frac{Z_\alpha - Z_{1-\beta}}{Z_{AQL} - Z_{RQL}} \right)^2$$ Z_α and $Z_{1-\beta}$

would have to be replaced with the quantiles of the t-distribution with $n-1$ degrees of freedom

k-method - lower specification limit, σ unknown

`find.plan()` uses an iterative approach to find n and k when σ is unknown.

k-method - lower specification limit, σ unknown

`find.plan()` uses an iterative approach to find n and k when σ is unknown.

```
find.plan(PRP=c(0.01, 0.95), CRP=c(0.046, 0.10), type="normal", s.type="unknown")
$n
[1] 63


$k
[1] 1.974026


$s.type
[1] "unknown"
```

k-method - lower specification limit, σ unknown

`find.plan()` uses an iterative approach to find n and k when σ is unknown.

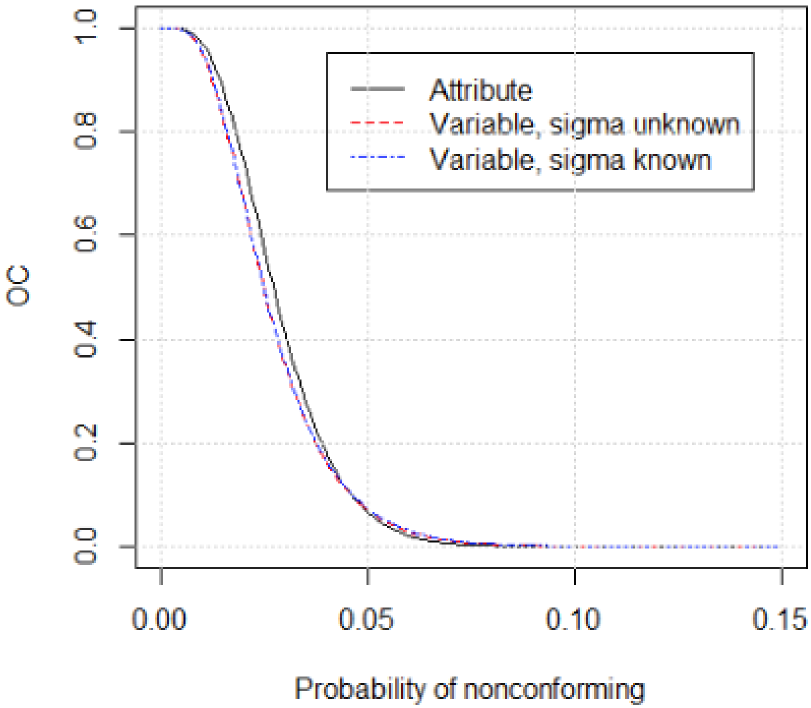
```
find.plan(PRP=c(0.01, 0.95), CRP=c(0.046, 0.10), type="normal", s.type="unknown")
$n
[1] 63


$k
[1] 1.974026


$s.type
[1] "unknown"
```

The sample size n for this plan with σ unknown is still much less than the $n = 172$ that would be required for an attribute plan with an equivalent OC curve

Comparing OC for an Attribute or Variable plan with σ known or unknown

```
library(AcceptanceSampling)
plnA<-OC2c(n=172,c=4,type="binomial",pd=seq(0,.15,.001))
plnVku<-OCvar(n=63,k=1.974026,pd=seq(0,.15,.001),s.type="unknown")
plnVkm<-OCvar(n=21,k=1.96411,pd=seq(0,.15,.001),s.type="known")
#Plot all three OC curves on the same graph
pd<-seq(0,.15,.001)
plot(pd,plnA@paccept,type='l',lty=1,col=1,xlab='Probability of
nonconforming',ylab='OC')
lines(pd,plnVku@paccept,type='l',lty=2,col=2)
lines(pd,plnVkm@paccept,type='l',lty=4,col=4)
legend(.04,.95,c("Attribute","Variable, sigma unknown","Variable, sigma
known"),lty=c(1,2,4),
       + col=c(1,2,4))
grid()
```

Comparing OC for an Attribute or Variable plan with σ known or unknown



Procedure for Variable Plans with σ unknown

When the standard deviation is unknown, conducting the sampling plan on a lot of material consists of the following steps:

1. Take a random sample of n items from the lot
2. Measure the critical characteristic x on each sampled item
3. Calculate the mean measurement $\overline{x}$, and the sample standard deviation s
4. Compare $(\overline{x} - LSL)/s$ to the acceptance constant k
5. If $(\overline{x} - LSL)/s > k$, accept the lot, otherwise reject the lot.

Example 2

The AQL is 0.01 or 1%, the RQL is 0.06 or 6%, the producers risk is $\alpha = 0.05$, the consumers risk $\beta = .10$, and the LSL=225psi.

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The AQL is 0.01 or 1%, the RQL is 0.06 or 6%, the producers risk is $\alpha = 0.05$, the consumers risk $\beta = .10$, and the LSL=225psi.

```
library(AcceptanceSampling)

find.plan(PRP=c(.01,.95), CRP=c(.06,.10), type='normal',s.type='unknown')

$n

[1] 42


$k

[1] 1.905285


$s.type

[1] "unknown"
```

Example 2

The AQL is 0.01 or 1%, the RQL is 0.06 or 6%, the producers risk is $\alpha = 0.05$, the consumers risk $\beta = .10$, and the LSL=225psi.

```
library(AcceptanceSampling)

find.plan(PRP=c(.01,.95), CRP=c(.06,.10), type='normal',s.type='unknown')

$n

[1] 42


$k

[1] 1.905285


$s.type

[1] "unknown"
```

After a sample of 42 with $\overline{x} = 255$ and $s = 15$, accept the lot since

$$Z_L = \frac{\overline{x} - LSL}{s} = \frac{255 - 225}{15} = 2.0 > 1.905285 = k$$

k-method - upper specification limit, σ known

When σ is known and there is an upper rather than lower specification limit, change steps 4 and 5

k-method - upper specification limit, σ known

When σ is known and there is an upper rather than lower specification limit, change steps 4 and 5

4. Compare $(\overline{x} - LSL)/\sigma$ to the acceptance constant $k = 1.967411$
5. If $(\overline{x} - LSL)/\sigma > k$, accept the lot, otherwise reject the lot.

$$\Downarrow$$

4. Compare $(USL - \overline{x})/\sigma$ to the acceptance constant k
5. If $(USL - \overline{x})/\sigma > k$, accept the lot, otherwise reject the lot.

k-method - upper specification limit, σ known

When σ is known and there is an upper rather than lower specification limit, change steps 4 and 5

4. Compare $(\overline{x} - LSL)/\sigma$ to the acceptance constant $k = 1.967411$
5. If $(\overline{x} - LSL)/\sigma > k$, accept the lot, otherwise reject the lot.

$$\Downarrow$$

4. Compare $(USL - \overline{x})/\sigma$ to the acceptance constant k
5. If $(USL - \overline{x})/\sigma > k$, accept the lot, otherwise reject the lot.

When σ is unknown change σ to s in steps 4. and 5. Again the appropriate sample size and acceptance constant would be found with the `find.plan()` function.

The M method, σ known, lower specification limit

For a variables sampling plan the M-method compares the estimated proportion below the LSL to a maximum allowable proportion.

The M method, σ known, lower specification limit

For a variables sampling plan the M-method compares the estimated proportion below the LSL to a maximum allowable proportion.

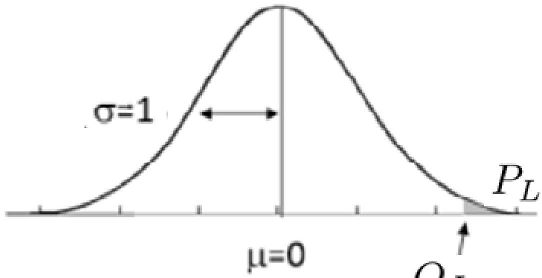
The estimated proportion below the LSL is is:

$$P_L = \int_{Q_L}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt, \quad \text{where} \quad Q_L = Z_L \left(\sqrt{\frac{n}{n-1}} \right), \quad \text{and} \quad Z_L = (LSL - \bar{x})/\sigma.$$

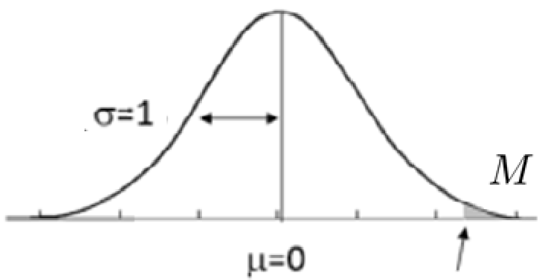
and the maximum allowable proportion defective is:

$$M = \int_{k\sqrt{\frac{n}{n-1}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt,$$

The M method, σ known, lower specification limit



$Q_L = Z_L\left(\sqrt{\frac{n}{n-1}}\right)$ $\qquad Z_L = (LSL - \overline{x})/\sigma$



$k\sqrt{\frac{n}{n-1}}$

Example 3

Reconsider Example 1, where k=1.6094, LSL=100, σ=8, and $\overline{x}$=110.

Example 3

Reconsider Example 1, where k=1.6094, LSL=100, σ=8, and $\overline{x}$=110.

$$Q_L = \left(\frac{110-100}{8}\right)\left(\sqrt{\frac{10}{9}}\right) = 1.3176. \quad P_L = \int_{1.3176}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \texttt{1-pnorm(1.3176)} = 0.0938.$$

Example 3

Reconsider Example 1, where k=1.6094, LSL=100, σ=8, and $\overline{x}$=110.

$$Q_L = \left(\frac{110-100}{8}\right)\left(\sqrt{\frac{10}{9}}\right) = 1.3176. \quad P_L = \int_{1.3176}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \texttt{1-pnorm(1.3176)} = 0.0938.$$

$$M = \int_{1.6094\sqrt{\frac{10}{9}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \texttt{1-pnorm(1.6094*sqrt(10/9))} = 0.0449.$$

Example 3

Reconsider Example 1, where k=1.6094, LSL=100, σ=8, and $\overline{x}$=110.

$$Q_L = \left(\frac{110-100}{8}\right)\left(\sqrt{\frac{10}{9}}\right) = 1.3176. \quad P_L = \int_{1.3176}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \texttt{1-pnorm(1.3176)} = 0.0938.$$

$$M = \int_{1.6094\sqrt{\frac{10}{9}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \texttt{1-pnorm(1.6094*sqrt(10/9))} = 0.0449.$$

Reject the lot since $P_L > M$

Example 3 continued

M and P_L can be calculated with **pnorm** or alternatively with the **MPn** and **EPn** functions in the **R** package **AQLSchemes** as shown below:

```
M<-pnorm(1.6094*sqrt(10/9),lower.tail=F)

PL<-pnorm(((110-100)/8)*sqrt(10/9),lower.tail=F)

M

[1] 0.04489973

PL

[1] 0.09381616

library(AQLSchemes)

PL<-EPn(sided="one",stype="known",LSL=100,sigma=8,xbar=110,n=10)

PL

[1] 0.09381616

M<-MPn(k=1.6094,n=10,stype="known")

M

[1] 0.04489973
```

The M method, σ unknown, lower specification limit

When the σ is unknown, the symmetric standardized Beta distribution is used rather than the standard normal distribution in computing P_L and M

$$B_x(a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x \nu^{a-1}(1-\nu)^{b-1} d\nu \quad = \texttt{pbeta(x,a,b)}$$

$$\hat{p}_L = B_x(a,b),$$

$$x = \max\left(0, .5 - .5 Q_L \left(\frac{\sqrt{n}}{n-1}\right)\right)$$

$$a = b = \tfrac{n}{2} - 1,$$

$$Q_L = \frac{\bar{x} - LSL}{s}$$

$$M = B_{B_M}\left(\frac{n-2}{2}, \frac{n-2}{2}\right)$$

$$M = B_{B_M}\left(\frac{n-2}{2}, \frac{n-2}{2}\right)$$

$$B_M = .5\left(1 - k\frac{\sqrt{n}}{n-1}\right)$$

Example 4

Reconsider Example 2, $n = 42$, $k = 1.905285$, $LSL = 225$, $\overline{x} = 255$, and $s = 15$.

Example 4

Reconsider Example 2, $n = 42$, $k = 1.905285$, $LSL = 225$, $\overline{x} = 255$, and $s = 15$.

$$a = b = \frac{42}{2} - 1 = 20,$$

$$\hat{p}_L = B_x(a, b) = \texttt{pbeta(.3419322,20,20)} = 0.02069563$$

$$B_M = .5\left(1 - 1.905285\left(\frac{\sqrt{42}}{42-1}\right)\right) = 0.3494188$$

$$M = B_{B_M}\left(\frac{42-2}{2}, \frac{42-2}{2}\right) = \texttt{pbeta(.3494188,20,20)} = 0.02630455,$$

Example 4

Reconsider Example 2, $n = 42$, $k = 1.905285$, $LSL = 225$, $\overline{x} = 255$, and $s = 15$.

$$a = b = \frac{42}{2} - 1 = 20,$$

$$\hat{p}_L = B_x(a, b) = \texttt{pbeta(.3419322,20,20)} = 0.02069563$$

$$B_M = .5\left(1 - 1.905285\left(\frac{\sqrt{42}}{42-1}\right)\right) = 0.3494188$$

$$M = B_{B_M}\left(\frac{42-2}{2}, \frac{42-2}{2}\right) = \texttt{pbeta(.3494188,20,20)} = 0.02630455,$$

Accept the lot since $\hat{p}_L < M$

The M method, σ unknown, lower specification limit

Calculating the estimated proportion below LSL, $\hat{p}_L$ and the maximum allowable proportion nonconforming, M, can simplified using the **EPn** and **MPn** functions in the **AQLSchemes** package as shown below:

```
library(AQLSchemes)
PL<-EPn(sided="one",stype="unknown",LSL=225,xbar=255,s=15,n=42)
PL
[1] 0.02069563
M<-MPn(k=1.905285,stype="unknown",n=42)
M
[1] 0.02630455
```

The M method, σ known, upper specification limit

Acceptance region is $P_U < M$, where

$$P_U = \int_{Q_U}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt.$$

$$Q_U = Z_U \left(\sqrt{\frac{n}{n-1}} \right) \quad Z_U = (USL - \bar{x})/\sigma.$$

The M method, σ unknown, upper specification limit

Acceptance region is $\hat{p}_U < M$, where

$$\hat{p}_U = B_x(a, b),$$

$$a = b = \frac{n}{2} - 1,$$

$$x = \max\left(0, .5 - .5Q_U\left(\frac{\sqrt{n}}{n-1}\right)\right),$$

$$Q_U = \frac{USL - \bar{x}}{s},$$

The M method, upper specification limit and lower specification limits

When σ is known the acceptance criterion is $P = (P_L + P_U) < M$, where P_L, P_U, and M are defined above.

The M method, upper specification limit and lower specification limits

When σ is known the acceptance criterion is $P = (P_L + P_U) < M$, where P_L, P_U, and M are defined above.

When σ is unknown the acceptance criterion is $\hat{p} = (\hat{p}_L + \hat{p}_U) < M$, where $\hat{p}_L$ and $\hat{p}_U$ are defined above,

Example 5

Consider the previous examples where $USL = 100$, $LSL = 90$. When $\sigma = 2.0$ was known, $n = 21$ and $k = 1.967411$. If $\overline{x} = 96.68$ then

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Consider the previous examples where $USL = 100$, $LSL = 90$. When $\sigma = 2.0$ was known, $n = 21$ and $k = 1.967411$. If $\overline{x} = 96.68$ then

$$Q_U = \left(\frac{(100 - 96.68)}{2.0}\right)\sqrt{\frac{21}{20}} = 1.701,$$

$$P_U = \int_{1.701}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 0.04447.$$

$$Q_L = \left(\frac{(96.68 - 90)}{2.0}\right)\sqrt{\frac{21}{20}} = 3.4225,$$

$$P_L = \int_{3.4225}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 0.00031.$$

$$M = \int_{1.967411\sqrt{\frac{21}{20}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 0.0219.$$

Example 5

Consider the previous examples where $USL = 100$, $LSL = 90$. When $\sigma = 2.0$ was known, $n = 21$ and $k = 1.967411$. If $\overline{x} = 96.68$ then

$$Q_U = \left(\frac{(100 - 96.68)}{2.0}\right)\sqrt{\frac{21}{20}} = 1.701,$$

$$P_U = \int_{1.701}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 0.04447.$$

$$Q_L = \left(\frac{(96.68 - 90)}{2.0}\right)\sqrt{\frac{21}{20}} = 3.4225,$$

$$P_L = \int_{3.4225}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 0.00031.$$

$$M = \int_{1.967411\sqrt{\frac{21}{20}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 0.0219.$$

Reject the lot since $P = (P_L + P_U) > M$

Example 5 continued

P and M can again be calculated using the `EPn` and `MPn` functions in the `AQLSchemes` package as shown below.

```
library(AQLSchemes)
# sigma known
P<-EPn(sided="two",stype="known",sigma=2,LSL=90,USL=100,xbar=96.68,n=21)
P
[1] 0.04478233
M<-MPn(k=1.967411,stype="known",n=21)
M
[1] 0.02190018
```

Example 5 continued

If σ was unknown in this earlier example, the sample size and acceptance constant (determined by the `find.plan` function) were $n = 63$ and $k = 1.97403$. Again the symmetric standardized Beta distribution is used rather than the standard normal distribution in computing $\hat{p}_L$, $\hat{p}_U$ and M. If $\overline{x} = 97.006$ after a sample of 63, then

Example 5 continued

If σ was unknown in this earlier example, the sample size and acceptance constant (determined by the `find.plan` function) were $n = 63$ and $k = 1.97403$. Again the symmetric standardized Beta distribution is used rather than the standard normal distribution in computing $\hat{p}_L$, $\hat{p}_U$ and M. If $\overline{x} = 97.006$ after a sample of 63, then

$$\hat{p}_U = B_x(a, b) = 0.06407,$$

$$a = b = \frac{63}{2} - 1 = 30.5,$$

$$x = \max\left(0, .5 - .5Q_U\left(\frac{\sqrt{63}}{63-1}\right)\right) = 0.4031,$$

$$Q_U = \frac{100 - 97.006}{1.9783} = 1.51342.$$

Example 5 continued

$$\hat{p}_L = B_x(a,b) = 0.000095,$$

$$a = b = \frac{63}{2} - 1 = 30.5,$$

$$x = \max\left(0, .5 - .5Q_L\left(\frac{\sqrt{63}}{63-1}\right)\right) = 0.2733,$$

$$Q_L = \frac{97.006 - 90}{1.9783} = 3.541.$$

$$M = B_{B_M}\left(\frac{63-2}{2}, \frac{63-2}{2}\right) = 0.02284,$$

$$B_M = .5\left(1 - 1.97403\frac{\sqrt{63}}{63-1}\right) = 0.37364.$$

Example 5 continued

Since $\hat{p} = (\hat{p}_L + \hat{p}_U) = 0.06416 > 0.02284$ reject the lot. These calculations can again be automated with the **`EPn`** and **`MPn`** functions.

```
library(AQLSchemes)

# sigma unknown

P<-EPn(sided="two",stype="unknown",LSL=90,USL=100,xbar=97.006,s=1.9783,n=63)

P

[1] 0.06416326

M<-MPn(k=1.97403,stype="unknown",n=63)

M

[1] 0.02284391
```

ANSI/ASQ Z1.9 Variable Sampling Scheme

ANSI/ASQ Z1.9 is the American national standard that replaced MIL-STD-414. It matches the OC performance of the ANSI/ASQ Z1.4 Attribute plans for the same inspection level, lot size and AQL. Therefore it is possible to switch back and forth between attribute and variable sampling plans using these two standards and retain the same operating characteristic.

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This variable sampling scheme is meant to be used when sampling a stream of lots from a supplier. They include normal, tightened, and reduced sampling plans and use the same switching rules as the ANSI/ASQ Z1.4 Attribute plans discussed in chapter 2.

ANSI/ASQ Z1.9 Variable Sampling Scheme

ANSI/ASQ Z1.9 is the American national standard that replaced MIL-STD-414. It matches the OC performance of the ANSI/ASQ Z1.4 Attribute plans for the same inspection level, lot size and AQL. Therefore it is possible to switch back and forth between attribute and variable sampling plans using these two standards and retain the same operating characteristic.

This variable sampling scheme is meant to be used when sampling a stream of lots from a supplier. They include normal, tightened, and reduced sampling plans and use the same switching rules as the ANSI/ASQ Z1.4 Attribute plans discussed in chapter 2.

The switching rules must be followed to gain the full benefit of the scheme which will result in greater protection for both supplier and customer.

ANSI/ASQ Z1.9 Variable Sampling Scheme

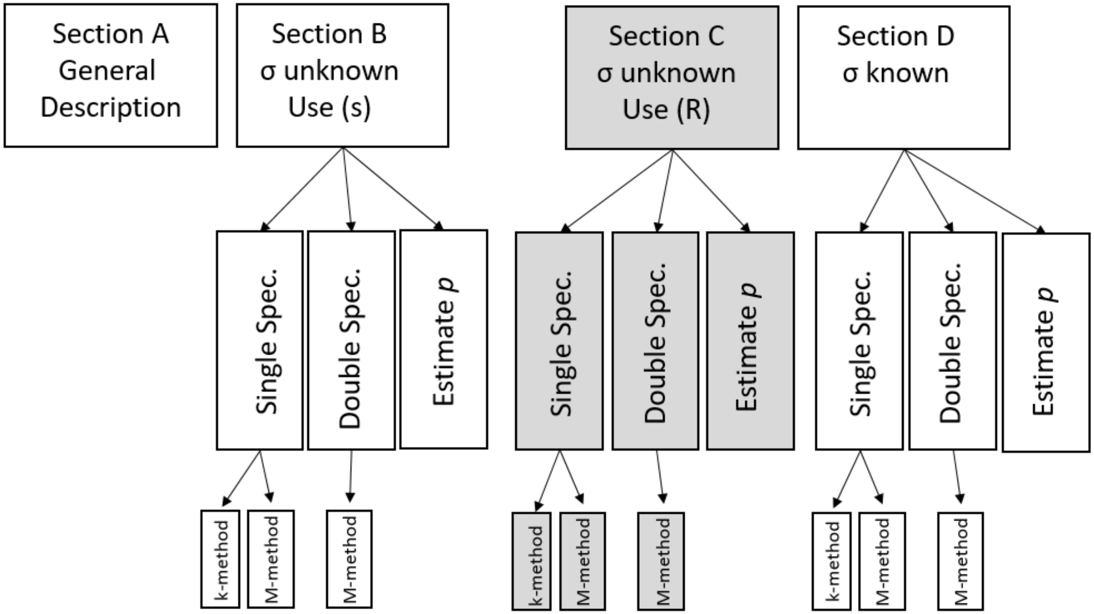


Figure 1: Content of MIL-STD 414

ANSI/ASQ Z1.9 Variable Sampling Scheme

The function `AAZ19()` in the R package `AQLSchemes` can retrieve the normal, tightened or reduced sampling plans for the variability known or unknown cases from the ANSI/ASQ Z1.9 standard.

ANSI/ASQ Z1.9 Variable Sampling Scheme

The function `AAZ19()` in the R package `AQLSchemes` can retrieve the normal, tightened or reduced sampling plans for the variability known or unknown cases from the ANSI/ASQ Z1.9 standard.

The function `AAZ19()` has two required arguments, the first argument `type`, that can take on the values `'Normal'`, `'Tightened'` or `'Reduced'`. It must be supplied to override the default value `'Normal'`. A second argument `stype` can take on the values `'unknown'` or `'known'`. It must be supplied to override the default value `'unknown'`. The function `AAZ19()` is called in the same the way the functions `AASingle()` and `AADouble()` were called. They were illustrated in the last chapter.

ANSI/ASQ Z1.9 Variable Sampling Scheme

The section of R code below and on the next two pages illustrates the function call, the interactive queries and answers and the resulting plan. The second argument was left out to get the default value.

```
> library(AQLSchemes)
> AAZ19('Normal')

What is the Inspection Level?

1: S-3
2: S-4
3: I
4: II
5: III

Selection: 4
```

ANSI/ASQ Z1.9 Variable Sampling Scheme

```
What is the Lot Size?


 1: 2-8                2: 9-15               3: 16-25              4: 26-50
 5: 51-90              6: 91-150             7: 151-280            8: 281-400
 9: 401-500           10: 501-1200          11: 1201-3200         12: 3201-10,000
13: 10,001-35,000     14: 35,001-150,000    15: 150,001-500,000   16: 500,001 and over


Selection: 4
```

ANSI/ASQ Z1.9 Variable Sampling Scheme

```
What is the AQL in percent nonconforming per 100 items?

 1: 0.10   2: 0.15   3: 0.25   4: 0.40   5: 0.65   6: 1.0    7: 1.5    8: 2.5    9: 4.0
10: 6.5   11: 10

Selection: 6
Sample size n =  5
Acceptability constant k =  1.524668
Maximum proportion non-conforming M =  0.0333
```

ANSI/ASQ Z1.9 Variable Sampling Scheme

```
What is the AQL in percent nonconforming per 100 items?

 1: 0.10   2: 0.15   3: 0.25   4: 0.40   5: 0.65   6: 1.0    7: 1.5    8: 2.5    9: 4.0
10: 6.5   11: 10

Selection: 6
Sample size n =  5
Acceptability constant k =  1.524668
Maximum proportion non-conforming M =  0.0333
```

The result shows that the sampling plan consists of taking a sample of 5 devices from the lot of 40 and comparing the estimated proportion non-conforming to 0.0333.

ANSI/ASQ Z1.9 Variable Sampling Scheme

If the operating temperatures of 5 sampled devices were (197,188,184,205, and 201). The `EPn()` function can be called to calculate the estimated proportion non-conforming as shown in the R code below.

```
library(AQLSchemes)

sample<-c(197,188,184,205,201)

EPn(sample,sided="two",LSL=180,USL=209)

Estimated proportion non-conforming =  0.02799209
```

ANSI/ASQ Z1.9 Variable Sampling Scheme

If the operating temperatures of 5 sampled devices were (197,188,184,205, and 201). The `EPn()` function can be called to calculate the estimated proportion non-conforming as shown in the R code below.

```
library(AQLSchemes)
sample<-c(197,188,184,205,201)
EPn(sample,sided="two",LSL=180,USL=209)
Estimated proportion non-conforming =  0.02799209
```

The argument `sample` in the function call is the vector of sample data values. The argument `sided` can be equal to `"two"` or `"one"`, depending on whether there are double specification limits or a single specification limit. Finally, the arguments `LSL` and `USL` give the specification limits. If there is only a lower specification limit, change `sided="two"` to `sided="one"` and leave out `USL`.

ANSI/ASQ Z1.9 Variable Sampling Scheme

$P = 0.02799209 < 0.0333 = M$, therefore accept the lot. If the sample mean, sample standard deviation, and the sample size have already been calculated and stored in the variables **`xb`**, **`sd`**, and **`ns`**, then the function call can also be given as

```
EPn(sided="two",LSL=180,USL=209,xbar=xb,s=sd,n=ns)
```

ANSI/ASQ Z1.9 Variable Sampling Scheme

The tightened sampling plan for the same inspection level, lot size, and AQL is found with the call:

```
> library(AQLSchemes)
> AAZ19('Tightened')
```

ANSI/ASQ Z1.9 Variable Sampling Scheme

The tightened sampling plan for the same inspection level, lot size, and AQL is found with the call:

```
> library(AQLSchemes)
> AAZ19('Tightened')
```

Answering the queries the same way as shown above results in the plan

```
Sample size n =  5
Maximum proportion non-conforming M =  0.0134
```

ANSI/ASQ Z1.9 Variable Sampling Scheme

The reduced sampling plan for the same inspection level, lot size, and AQL is found with the call:

```
> library(AQLSchemes)
> AAZ19('Reduced')
```

ANSI/ASQ Z1.9 Variable Sampling Scheme

The reduced sampling plan for the same inspection level, lot size, and AQL is found with the call:

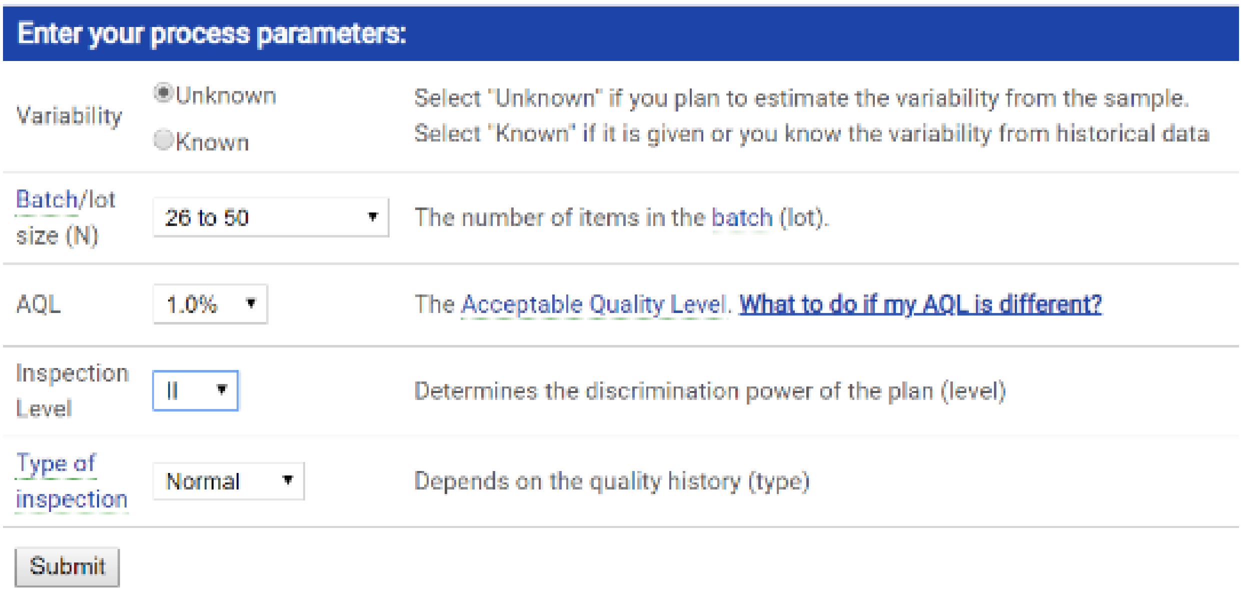
```
> library(AQLSchemes)
> AAZ19('Reduced')
```

Answering the queries the same way as shown above results in the plan

```
Sample size n =  4
Maximum proportion non-conforming M =  0.0550
```

ANSI/ASQ Z1.9 Variable Sampling Scheme

The website [sqc online calculator](https://www.sqconline.com/) also provides online calculators for recalling ANSI/ASQ Standard Z1.9 plans. To repeat the last example go to the website and log in, then click on the Acceptance Sampling menu. On the next page, click on the MIL-STD-414 ANSI/ASQ Z1.9 menu. Fill out the template that pops up as shown below and click the Submit button to create the sampling plan.



ANSI/ASQ Z1.9 Variable Sampling Scheme

The result is shown below. Filling out the template below the resulting sampling plan calculates the estimated proportion nonconforming (like the R function `EPn`.

ANSI/ASQC Z1.9 Tables

For a lot of **26 to 50** items, and **AQL= 1.0%** , with inspection level **II** , the **Normal** inspection plan is:

Sample **5*** items.

If the estimated percent of non-conforming (defective) items is
3.33% or less --> accept the lot.
Otherwise, reject it.

Note: This sampling plan will yield valid results only if applied with the Z1.9 switching rules.

To estimate the percent of non-conforming items in your process, take a sample of size **5** and enter the values into the next table.

To estimate your process % non-conforming (defectives) enter:

Sample Average ($\bar{x}$):	195	The average of the 5 measurements
Process Standard Deviation (s):	8.803	The standard deviation of the 5 measurements
Lower Specification Limit:	180	The smallest value for your measurement that is considered acceptable. Leave blank if there is no lower limit.
Upper Specification Limit:	209	The largest value for your measurement that is considered acceptable. Leave blank if there is no upper limit.

Submit

Measurement Error

Repeated measurements of the same part or process output do not always result in exactly the same value. This is called measurement error. It is necessary to estimate the variability in measurement error in order to determine if the measurement process is accurate enough for sampling inspection and process monitoring and control.

Measurement Error

Repeated measurements of the same part or process output do not always result in exactly the same value. This is called measurement error. It is necessary to estimate the variability in measurement error in order to determine if the measurement process is accurate enough for sampling inspection and process monitoring and control.

Gauge capability or Gauge R&R studies are conducted to estimate the magnitude of measurement error and partition this variability into its sources. The major sources of measurement error are repeatability and reproducibility

Gauge R&R studies

- Repeatability refers to the error in measurements that occur when the same operator uses the gauge or measuring device to measure the same part or process output repeatedly.

Gauge R&R studies

- Repeatability refers to the error in measurements that occur when the same operator uses the gauge or measuring device to measure the same part or process output repeatedly.
- Reproducibility refers to the measurement error that occurs due to different measuring conditions such as the operator making the measurement, or the environment where the measurement is made.

Gauge R&R studies

A basic gauge R&R study is conducted by having random sample of several operators measure each part or process output in a sample repeatedly. The operators are blinded as to which part they are measuring at the time they measure it. The data in Table 3.1 is from a study where each of 3 operators measured 10 samples twice each.

TABLE 3.1: Results of Gauge R&R Study

Sample	Operator 1	Operator 2	Operator 3
1	103.24	103.16	102.96
2	103.92	103.81	103.76
3	109.13	108.86	108.70
4	108.35	108.11	107.94
5	105.51	105.06	104.84
6	106.63	106.61	106.60
7	109.29	108.96	108.84
8	108.76	108.39	108.23
9	108.03	107.86	107.72
10	106.61	106.32	106.21
1	103.56	103.26	103.01
2	103.86	103.80	103.75
3	109.23	108.79	108.75
4	108.29	108.24	107.99
5	105.53	105.11	104.80
6	106.65	106.57	106.55
7	109.28	109.12	109.03
8	108.72	108.43	108.27
9	108.11	107.84	107.79
10	106.77	106.23	106.13

Gauge R&R studies

An analysis of variance is used to to estimate σ_p^2, the variance among different parts (or in this study samples); σ_o^2, the variance among operators; σ_{po}^2,the variance among part by operator; and σ_r^2, the variance due to repeat measurements on one part (or sample in this study) by one operator.

Gauge R&R studies

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The gauge repeatability variance is defined to be σ_r^2, and the gauge reproducibility variance is defined to be $\sigma_o^2 + \sigma_{po}^2$.

Gauge R&R studies

The `gageRRDesign()` function in the R package `QualityTools` can produce a design for a Gauge R&R study. The code below creates the design in nonrandom order and the measurement data from Table 3.1 is assigned to the response.

```
library(qualityTools)
design=gageRRDesign(Operators=3, Parts=10 ,
  Measurements=2, randomize=FALSE)
#set the response
response(design)=c(103.24,103.16,102.96,103.92,103.81,
                   103.76,109.13,108.86,108.70,108.35,
                   108.11,107.94,105.51,105.06,104.84,
                   106.63,106.61,106.60,109.29,108.96,
                   108.84,108.76,108.39,108.23,108.03,
                   107.86,107.72,106.61,106.32,106.21,
                   103.56,103.26,103.01,103.86,103.80,
                   103.75,109.23,108.79,108.75,108.29,
                   108.24,107.99,105.53,105.11,104.80,
                   106.65,106.57,106.55,109.28,109.12,
                   109.03,108.72,108.43,108.27,108.11,
                   107.84,107.79,106.77,106.23,106.13)
```

Gauge R&R studies

The `gageRR(design)` function in the `QualityTools` will produce the analysis of variance and the various components of variance. The argument `design` supplied in the function call was created in the code on the last slide.

```
gA<-gageRR(design)

##
## AnOVa Table -  crossed Design
##                Df Sum Sq Mean Sq    F value   Pr(>F)
## Operator        2    1.49   0.744   155.740  < 2e-16 ***
## Part            9  241.85  26.873 5627.763  < 2e-16 ***
## Operator:Part  18    0.35   0.019    4.072 0.000346 ***
## Residuals      30    0.14   0.005
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## ----------
##
## Gage R&R
##                  VarComp VarCompContrib  Stdev StudyVar StudyVarContrib
## totalRR          0.04832        0.01068 0.2198    1.319          0.1033
##  repeatability   0.00478        0.00106 0.0691    0.415          0.0325
##  reproducibility 0.04354        0.00963 0.2087    1.252          0.0981
##    Operator      0.03621        0.00800 0.1903    1.142          0.0895
##    Operator:Part 0.00733        0.00162 0.0856    0.514          0.0403
## Part to Part     4.47552        0.98932 2.1155   12.693          0.9946
## totalVar         4.52384        1.00000 2.1269   12.762          1.0000
##
## ---
##  * Contrib equals Contribution in %
##  **Number of Distinct Categories (truncated signal-to-noise-ratio) = 1
3
```

Gauge R&R studies

In the output on the last slide, it can be seen that the varaibility among parts, $\sigma_p^2 = 4.47552$, is over 98% of the total variability. The measurement error, `totalR&R` or $\sigma_g^2 = .04832$ is a sum of the repeatability plus reproducibility and is only 1.068% of the total variability.

Gauge R&R studies

In the output on the last slide, it can be seen that the varaibility among parts, $\sigma_p^2 = 4.47552$, is over 98% of the total variability. The measurement error, `totalR&R` or $\sigma_g^2 = .04832$ is a sum of the repeatability plus reproducibility and is only 1.068% of the total variability.

In this example the measurement error is very small compared to the actual variability among parts. When the measurement error is large relative to the variance among the parts, the repeatability and reproducibility components can help you determine where to focus efforts to reduce the measurement error.

Gauge R&R studies

Generally, the gauge or measuring instrument is considered to be suitable if the process to tolerance $P/T = \frac{6 \times \sigma_{gauge}}{USL - LSL} \leq 0.10$ where $\sigma_{gauge} = \sqrt{\sigma_g^2}$ and USL, and LSL are the upper and lower specification limits for the part being measured.

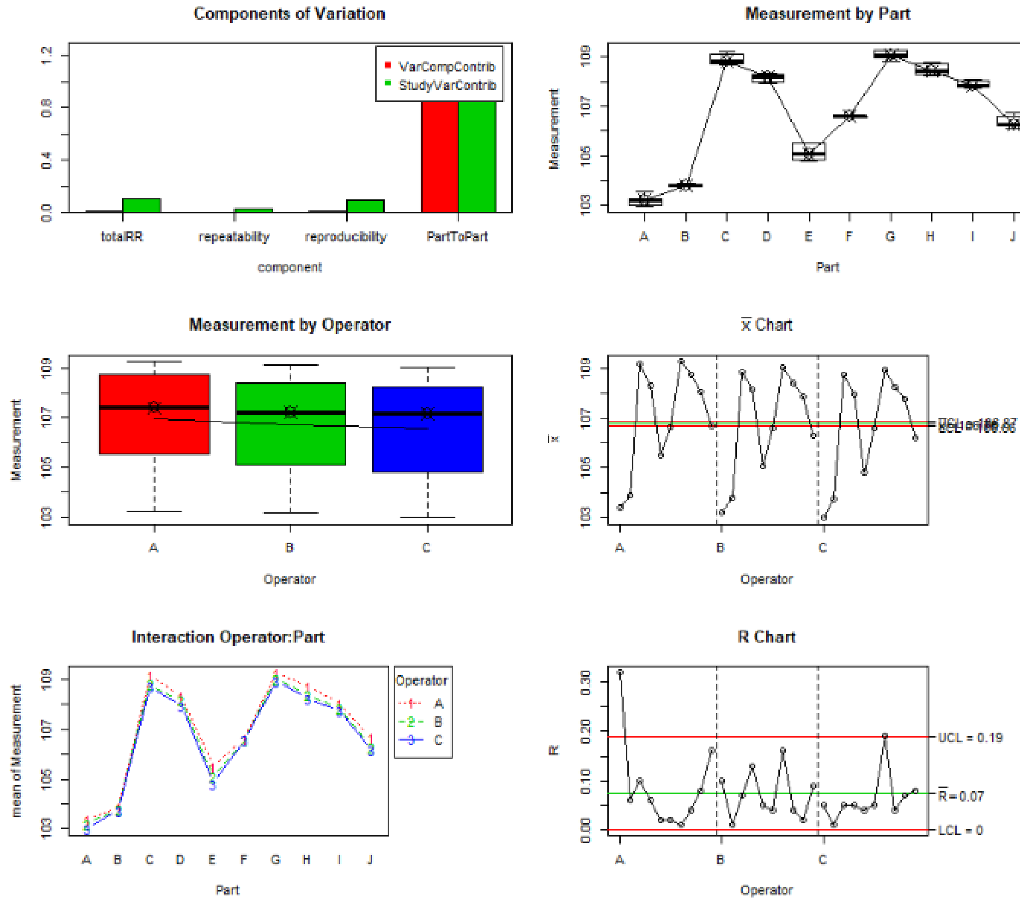
Gauge R&R studies

Generally, the gauge or measuring instrument is considered to be suitable if the process to tolerance $P/T = \frac{6 \times \sigma_{gauge}}{USL - LSL} \leq 0.10$ where $\sigma_{gauge} = \sqrt{\sigma_g^2}$ and USL, and LSL are the upper and lower specification limits for the part being measured.

Otherwise the measurements will not be accurate enough to determine whether the specification limits are met.

Gauge R&R studies

Graphical summary of gauge R&R study from `plot(gA)` function



Summary

This chapter has discussed variables sampling plans and schemes. The major advantage to variables sampling plans over attribute plans is the same protection levels with reduced sample sizes. The table below (patterned after one presented by (Schilling and Neubauer 2017) shows the average sample numbers for various plans that are matched to a single sampling plan for attributes with $n = 50$, $c = 2$. In addition to reduced sample sizes, variable plans provide information like the mean and estimated proportion defective below the lower specification limit and above the upper specification limit. This information can be valuable to the producer in correcting the cause of rejected lots and improving the process to produce at the AQL level or better.

Plan	Average Sample Number
Single Attributes	50
Double Attributes	43
Multiple Attributes	35
Variables (σ unknown)	27
Variables (σ known)	12

Summary

When a continuous stream of lots is being sampled, the published schemes with switching rules are more appropriate. They provide better protection for producer and consumer at a reduced average sample number. The variables plans and published tables described in this chapter are based on the assumption that the measured characteristic is normally distributed.

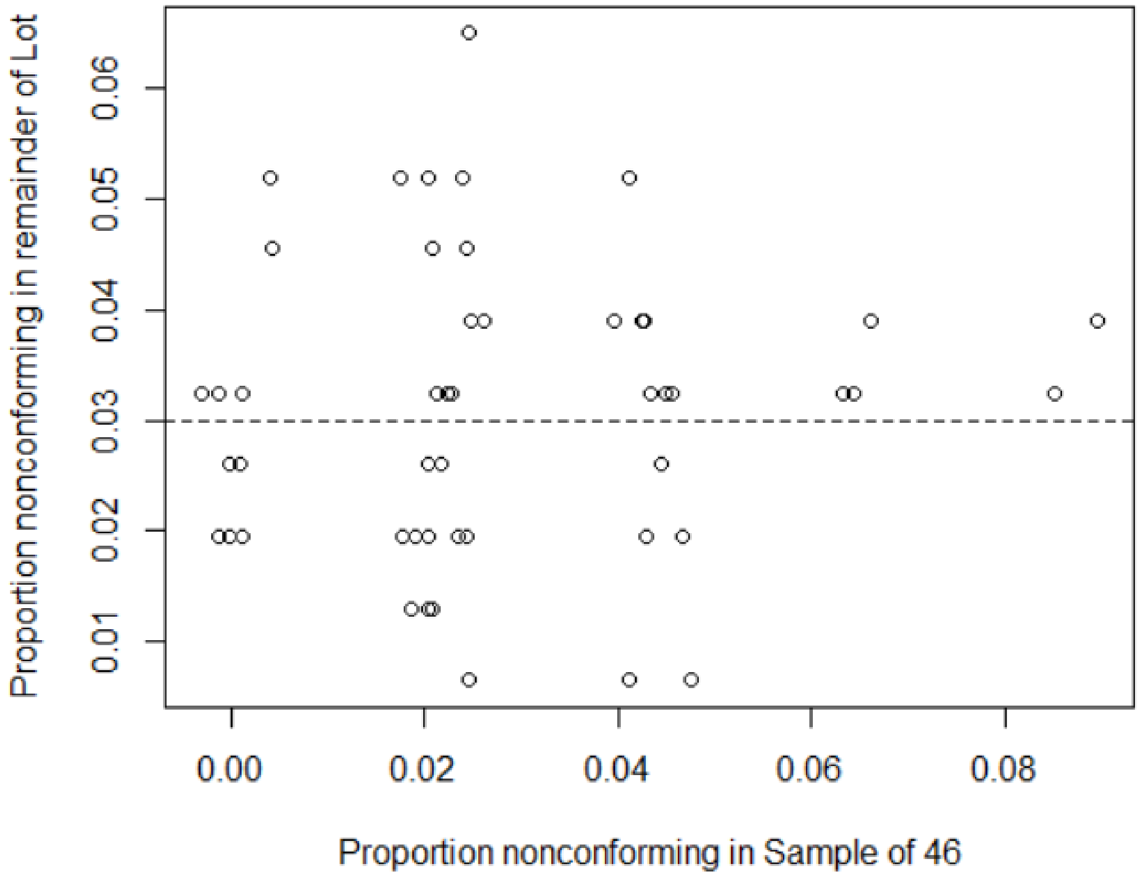
Summary

That being said, the need for any kind of acceptance sampling is dependent on the consistency of the supplier's process. If the supplier's process is consistent (or in a state of statistical control) and is producing defects or nonconformities at a level that is acceptable to the customer, (Deming 1986) pointed out that no inspection is necessary or cost effective. On the other hand, if the supplier's process is consistent but producing defects or nonconformities at a level that is too high for the customer to tolerate, 100% inspection should always be required. This is because the number (or proportion) nonconforming in a random sample from the lot is uncorrelated with the proportion non-conforming in the remainder of the lot.

Summary

```
# Lot size N=200 with an average 3% defective
p<-rbinom(50,200,.03)
r<-seq(1:50)
# This loop simulates the number non-conforming in a sample of 46
# items from each of the simulated lots
for (i in 1:length(p)) {
  r[i]<-rhyper(1,p[i],200-p[i],46)
}
ps<-r/46 # this statement calculates the proportion non-conforming in each lot
pr<-(p-r)/154 #This statement calculates the proportion non-conforming
# in the unsampled portion of each lot
plot(ps,pr,xlab='Proportion nonconforming in Sample of 46',
     ylab='Proportion nonconforming in remainder of Lot')
abline(h=(.03*46)/46,lty=2)
cor(ps,pr)
```

Summary



Proportion nonconforming in remainder of Lot
0.01
0.02
0.03
0.04
0.05
0.06
0.00
0.02
0.04
0.06
0.08
Proportion nonconforming in Sample of 46